

DEVELOPMENT OF SIMPLIFIED PROCEDURE FOR COMPUTING THE ABSORPTION OF SOUND BY THE ATMOSPHERE AND APPLICABILITY TO AIRCRAFT NOISE CERTIFICATION: PROPOSED SAE METHOD

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13. ABSTRACT (Maximum 200 words) This report presents the results of the study to extend the useful attenuation range of the Approximate Method outlined in the American National Standard, "Method for Calculation of the Absorption of Sound by the Atmosphere" (ANSI S1.26-1995), and provide a basis for replacing the current Society of Automotive Engineers Aerospace Recommended Practice 866A, "Standard Values of Atmospheric Absorption as a Function of Temperature and Humidity" (SAE ARP 866A). The report describes the implementation of the one-third octave-band adaptations of the ISO/ANSI pure-tone equations, and the development and testing of the proposed SAE Method.				
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METRIC/ENGLISH CONVERSION FACTORS

ENGLISH TO METRIC

LENGTH (APPROXIMATE)	
1 inch (in)	2.5 centimeters (cm)
1 foot (ft)	30 centimeters (cm)
1 yard (yd)	0.9 meter (m)
1 mile (mi)	1.6 kilometers (km)

AREA (APPROXIMATE)	
1 square inch (sq in, in ²)	6.5 square centimeters (cm ²)
1 square foot (sq ft, ft ²)	0.09 square meter (m ²)
1 square yard (sq yd, yd ²)	0.8 square meter (m ²)
1 square mile (sq mi, mi ²)	2.6 square kilometers (km ²)
1 acre = 0.4 hectare (he)	4,000 square meters (m ²)

MASS – WEIGHT (APPROXIMATE)	
1 ounce (oz)	28 grams (gm)
1 pound (lb)	0.45 kilogram (kg)
1 short ton = 2,000 pounds (lb)	0.9 tonne (t)

VOLUME (APPROXIMATE)	
1 teaspoon (tsp)	5 milliliters (ml)
1 tablespoon (tbsp)	15 milliliters (ml)
1 fluid ounce (fl oz)	30 milliliters (ml)
1 cup (c)	0.24 liter (l)
1 pint (pt)	0.47 liter (l)
1 quart (qt)	0.96 liter (l)
1 gallon (gal)	3.8 liters (l)
1 cubic foot (cu ft, ft ³)	0.03 cubic meter (m ³)
1 cubic yard (cu yd, yd ³)	0.76 cubic meter (m ³)

TEMPERATURE (EXACT)	
$[(x-32)(5/9)]$ °F	y °C

METRIC TO ENGLISH

LENGTH (APPROXIMATE)	
1 millimeter (mm)	0.04 inch (in)
1 centimeter (cm)	0.4 inch (in)
1 meter (m)	3.3 feet (ft)
1 meter (m)	1.1 yards (yd)
1 kilometer (km)	0.6 mile (mi)

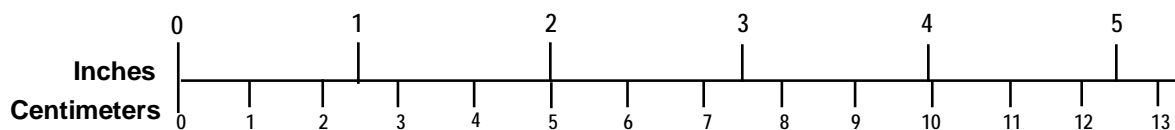
AREA (APPROXIMATE)	
1 square centimeter (cm ²)	0.16 square inch (sq in, in ²)
1 square meter (m ²)	1.2 square yards (sq yd, yd ²)
1 square kilometer (km ²)	0.4 square mile (sq mi, mi ²)
10,000 square meters (m ²)	1 hectare (ha) = 2.5 acres

MASS – WEIGHT (APPROXIMATE)	
1 gram (gm)	0.036 ounce (oz)
1 kilogram (kg)	2.2 pounds (lb)
1 tonne (t)	1,000 kilograms (kg)
	1.1 short tons

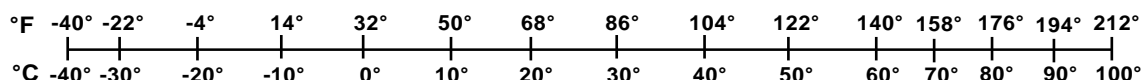
VOLUME (APPROXIMATE)	
1 milliliter (ml)	0.03 fluid ounce (fl oz)
1 liter (l)	2.1 pints (pt)
1 liter (l)	1.06 quarts (qt)
1 liter (l)	0.26 gallon (gal)
1 cubic meter (m ³)	36 cubic feet (cu ft, ft ³)
1 cubic meter (m ³)	1.3 cubic yards (cu yd, yd ³)

TEMPERATURE (EXACT)	
$[(9/5)y + 32]$ °C	x °F

QUICK INCH - CENTIMETER LENGTH CONVERSION



QUICK FAHRENHEIT - CELSIUS TEMPERATURE CONVERSION



For more exact and or other conversion factors, see NIST Miscellaneous Publication 286, Units of Weights and Measures. SD Catalog No. C13 10286

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1. INTRODUCTION

The United States Department of Transportation, John A. Volpe National Transportation Systems Center (Volpe Center) Environmental Measurement and Modeling Division, in support of the Federal Aviation Administration's (FAA) Office of Environment and Energy (AEE), and working under the auspices of the Society of Automotive Engineers' (SAE) Project Working Team (PWT) has completed a study of a proposed new method to modernize the requirements for calculating atmospheric absorption of sound by the atmosphere. The new method, referred to herein as the proposed SAE Method, utilizes the pure-tone sound absorption algorithms of two published standards, the International Standard, "Acoustics - Attenuation of Sound During Propagation Outdoors - Part 1: Calculation of the Absorption of Sound by the Atmosphere" (ISO 9613-1)² and the American National Standard, "Method for Calculation of the Absorption of Sound by the Atmosphere" (ANSI S1.26-1995)³. References 2 and 3 are herein referred to as ISO/ANSI^{2,3}.

This report presents the results of the study to extend the useful attenuation range of the Approximate Method³ outlined in the ANSI document, and provide a basis for replacing the current Society of Automotive Engineers Aerospace Recommended Practice 866A, "Standard Values of Atmospheric Absorption as a Function of Temperature and Humidity" (SAE ARP 866A)⁴. Section 1 is an introduction to the topic of atmospheric absorption as it relates to aircraft noise certification and states the objective of the study. Section 2 describes the implementation of the one-third octave-band adaptations of the ISO/ANSI^{2,3} pure-tone equations. Section 3 describes the development of the proposed SAE Method. Section 4 describes the sensitivity testing of the proposed SAE Method. Section 5 presents conclusions and recommendations.

1.1 Background

Aircraft noise certification in the United States is performed under the auspices of the Federal Aviation Regulation, Part 36, "Noise Standards: Aircraft Type and Airworthiness Certification" (Part 36)¹. The international counterpart to Part 36 is the International Civil Aviation Organization (ICAO) Annex 16⁵. Part 36 requires that aircraft position data, performance data and noise data be corrected to the following homogeneous, reference atmospheric conditions for noise certification:

- Sea level pressure of 2116 psf (76 cm of mercury, 101.325 kPa);
- Ambient temperature of 77 degrees Fahrenheit (25 degrees Celsius);
- Relative Humidity of 70 percent; and
- Zero wind.

An integral component of the Part 36 noise data correction process is the computation of sound absorption over the propagation path. Part 36 requires that atmospheric absorption as a function of propagation path distance be computed in one-third octave-bands from 50 Hz to 10 kHz using the method described in SAE ARP 866A⁴. Herein, this method is referred to as the SAE 866A Method. The SAE 866A Method includes an empirical means of adapting its pure-tone equations for use in a fractional octave-band analysis. Specifically, it states that for one-third octave-bands with mid-band frequencies at or below 4 kHz, the sound attenuation rates should be computed at the mid-band frequency of the nominal one-third octave frequency band; and for higher frequency bands, the lower-band edge-frequency should be used.

The two referenced published standards, ISO/ANSI^{2,3}, present theoretically-founded and experimentally-validated empirical algorithms for computing atmospheric absorption. While the SAE

866A absorption is a function only of temperature and humidity, the ISO/ANSI^{2,3} algorithms also include the effects of atmospheric pressure.

The ISO/ANSI^{2,3} equations for computing sound attenuation as a function of propagation distance are arithmetically identical to one another and specify computation as a function of temperature, relative humidity, and atmospheric pressure for *single, discrete frequencies or pure-tones*. However, Part 36 requires that noise data be analyzed in one-third octave-bands. Recognizing this fact, the authors of these two standards included methods for adapting the pure-tone algorithms for use in a fractional-octave-band analysis, e.g. one-third octave-band analysis.

Annex D of both the ISO² and ANSI³ standards present a relatively complex but technically sound method of adapting the pure-tone algorithms for use in a fractional-octave-band analysis. This method is referred to herein as the Spectrum Integration or “Exact Method.” The Exact Method requires knowledge of both the narrow-band characteristics of the sound source and the frequency response characteristics of the one-third octave-band filters used in the analysis. Due to these requirements, its use for aircraft noise certification is not realistic.

Annex E of the ANSI³ standard presents a more empirical method of adapting the pure-tone algorithms to one-third octave-bands, known as the “Approximate Method.” The Approximate Method does not require knowledge of the narrow-band characteristics of the sound source. It uses a second-order equation to approximate one-third octave-band level attenuation based on the frequency response characteristics of a third-order Butterworth filter⁶. As stated in the ANSI standard, it may be substituted for the Exact Method, with resulting errors of less than $\pm 5\%$, under conditions where the pure-tone absorption over the full path-length of any exact mid-band frequency is less than 50 dB. Consequently, the Approximate Method is not considered appropriate for the adjustment of aircraft noise certification data or for development of noise-power-distance data to be used in a computer model such as the FAA’s Integrated Noise Model (INM), for which absorption levels far greater than 50 dB are commonplace. The ISO standard does not present a method analogous to the Approximate Method of the ANSI standard.

The following definitions from the ISO/ANSI^{2,3} standards are repeated for clarity of the discussions presented herein:

- Case 1: One-third octave-band sound pressure levels known at the source are determined for a distant receiver.
- Case 2: One-third octave-band sound pressure levels known at the receiver are determined for the source.
- Case 3: One-third octave-band sound pressure levels known at a receiver location are adjusted for that receiver or to a new receiver location by accounting for differences in attenuation due to atmospheric absorption resulting from a different set of meteorological conditions along the propagation path.

Note: Cases 1 and 2 are used for propagation in a single direction. Whereas Case 3 is used for two-way propagation and is normally applicable to aircraft noise certification work.

Currently, the computation of atmospheric absorption for aircraft noise certification is performed using a two-step, **reciprocal** process (Case 3). First, absorption is computed for each one-third octave-band based on the temperature, humidity, and propagation distance at the time of the certification test (test-day absorption). Second, absorption is computed for each one-third octave-band based on the reference temperature, humidity, and the reference propagation distance (reference-day absorption). The as-measured noise data are then corrected to reference-day atmospheric conditions by arithmetically adding the test-day absorption, and then subtracting the reference-day absorption, taking into account differences in spherical spreading losses, as well as other physical effects. The process is

reciprocal in the sense that a user can take the reference-day results and work backward to recalculate the original test-day data.

This correction process is performed on a one-third octave-band basis, and the individual bands are later combined into required noise metrics, typically the sound exposure level (SEL), denoted by the symbol L_{AE} , or the effective perceived noise level (EPNL), denoted by the symbol L_{EPN} . For the purpose of this study, L_{PN} (perceived noise level) was used in place of L_{EPN} . The net result is a sound level adjusted to a specified reference distance, and a reference-day¹ temperature and relative humidity of 77 degrees Fahrenheit (25 degrees Celsius) and 70 percent (%RH), respectively.

All computations and comparisons in this report are made for a homogeneous atmosphere at a reference pressure of one standard atmosphere (101.325 kPa) unless otherwise noted.

1.2 Objective

The objectives of this study are: (1) to develop an empirical algorithm for calculating one-third octave-band attenuations utilizing the pure-tone sound absorption algorithms of the ISO/ANSI^{2,3} standards; (2) to simplify the computational process of the Exact Method; and (3) to extend the useful absorption range of the Approximate Method. The resultant approach would replace the current SAE 866A Method for correcting sound level data for specific atmospheric conditions.

1.3 Overview

An empirical algorithm (the proposed SAE Method) utilizing the pure-tone sound absorption algorithms of the ISO/ANSI^{2,3} standards was developed to simplify the computational process of the Exact Method, and is recommended for mid-band attenuation up to 500 dB.

Table 4 summarizes the one-third octave-band level difference data for the proposed SAE Method and the Exact Method, for a variety of environmental conditions, using a representative aircraft noise spectrum, and spanning an altitude range of 75 to 7620 meters. Tests indicate the method is usable for frequencies of 25 Hz to 20 kHz.

The proposed SAE Method is seen to be more accurate than the SAE 866A Method and, unlike the SAE 866A Method, it takes into account the effects of changes in atmospheric pressure on sound absorption, as well as the effects of temperature and relative humidity.

2. IMPLEMENTATION

This section describes the implementation of the one-third octave-band adaptations of the ISO/ANSI^{2,3} pure-tone equations.

The general ISO/ANSI^{2,3} equations for computing sound attenuation rates are arithmetically identical. These equations provide a means of computing attenuation rates at a single discrete frequency, i.e., for pure-tones. The common equations used for computing pure-tone sound attenuation by atmospheric absorption are shown in Equations [1] through [6]. The pure-tone equations are the foundation of the Exact Method, the Approximate Method, and the proposed SAE Method described herein.

The sound attenuation rate, α in decibels per meter, is computed as follows:

$$\begin{aligned}\alpha(f) = & 8.686f^2 \{ [1.84 \times 10^{-11} (p_a/p_r)^{-1} (T/T_r)^{1/2}] \\ & + (T/T_r)^{-5/2} \{ 0.01275 \exp(-2239.1/T) [f_{rO}/(f_{rO}^2 + f^2)] \\ & + 0.1068 \exp(-3352.0/T) [f_{rN}/(f_{rN}^2 + f^2)] \} \} \end{aligned} \quad [1]$$

where:

f = pure-tone frequency in Hz for which the sound attenuation rate is to be computed;

p_a = ambient atmospheric pressure in kPa
(test-day, reference-day, or representative-pathlength pressure, as appropriate);

p_r = 101.325 kPa, reference pressure of one standard atmosphere;

T = ambient atmospheric temperature in K
(test-day, reference-day, or representative pathlength temperature, as appropriate);

T_r = 293.15 K, reference ambient temperature;

$$f_{rO} = \frac{p_a}{p_r} \{ 24 + [(4.04 \times 10^4 h)(0.02 + h)] / (0.391 + h) \}; \quad [2]$$

$$f_{rN} = \frac{p_a}{p_r} (T/T_r)^{-1/2} \times (9 + 280h \exp\{-4.170[(T/T_r)^{-1/3} - 1]\}). \quad [3]$$

In Equations [2] and [3] for f_{rO} and f_{rN} , h is equivalent to the molar concentration of water vapor, as a percentage, and is computed as follows:

$$h = h_{\text{rel}}(p_{\text{sat}}/p_r)(p_a/p_r)^{-1} \quad [4]$$

where:

h_{rel} = relative humidity in percent
(test-day, reference-day, or representative-pathlength relative humidity, as appropriate);

$$p_{\text{sat}} = (p_r)10^V; \text{ and} \quad [5]$$

$$\begin{aligned} V = & 10.79586[1 - (T_{01}/T)] - 5.02808 \log_{10}(T/T_{01}) \\ & + 1.50474 \times 10^{-4} \{1 - 10^{-8.29692[(T/T_{01})-1]}\} \\ & + 0.42873 \times 10^{-3} \{-1 + 10^{4.76955[1-(T_{01}/T)]}\} - 2.2195983 \end{aligned} \quad [6]$$

where:

$$T_{01} = 273.16 \text{ K, triple-point isotherm temperature}$$

Although Equations [1] through [6] are common to the methods described herein, the procedure used for adapting the pure-tone attenuation rate for use in a one-third octave-band analysis on wideband sound is quite different and is described in the following sections, Sections 2.1 through 2.4.

2.1 Exact Method

The Exact Method, or Spectrum Integration Method, is a relatively complex, technically sound approach to adapting pure-tone sound absorption algorithms (see Equations [1] through [6]) for one-third octave-band analysis. Annex D of both the ISO² and ANSI³ standards provides general guidance for implementing this method, but leaves several parameters and assumptions to the discretion of the user. In addition to being open-ended, it also requires knowledge of both the narrow-band characteristics of the sound source and the frequency response characteristics of the one-third octave-band filters used in the analysis. Due to these requirements, it places an undue burden on applicants for aircraft certification, and its use for aircraft noise certification is not realistic. Therefore, the Exact Method is not considered a viable option for regulatory adoption. It is used herein as a point of reference because it is considered the most technically sound approach. The ISO² and ANSI³ documents also outline an approach for calculating attenuation effects on propagation through a stratified atmosphere in which attenuation is summed over discretized segments of the atmosphere. This technique is employed in the proposed SAE Method sensitivity analysis presented in Section 4.3. Specific assumptions made by the Volpe Center in implementing the Exact Method are discussed.

Rather than using the pure-tone attenuation at a single frequency to represent the average attenuation of the full one-third octave band, the Exact Method applies atmospheric absorption over narrow bands, for which attenuation is assumed constant. If spectral data are not available at the higher resolution of the narrow-bands, the narrow-band spectra can be interpolated from one-third octave-band data. In the Volpe implementation, the equivalent level representing the reference bandwidth of 1 Hz at a one-third octave-band's mid-band frequency, $L_S(f_{m,i})$, was calculated from the as-measured one-third octave-band sound pressure level, $L_{BS}(f_{m,i})$, as:

$$L_S(f_{m,i}) = L_{BS}(f_{m,i}) - 10 \log_{10}(B_i/B_0), \quad [7]$$

where:

$$L_{BS}(f_{m,i}) = \text{the as-measured sound pressure level for one-third octave-band } i;$$

$i = 17, \dots, 40$, integers representing one-third octave-bands from 50 Hz to 10 kHz;

$$f_{m,i} = 10^{i/10} \quad [8]$$

where:

$f_{m,i}$ is the exact mid-band frequency for one-third octave-band i , in Hz, for base-10 design one-third octave-band filters;

$B_i = 0.23077f_{m,i}$, the exact bandwidth for base-10 designed one-third octave-band filter i ;

$B_0 = 1$ Hz, normalizing bandwidth.

Note: B_0 defines the reference bandwidth for the derived pressure spectrum level data. A bandwidth of 1 Hz was selected for this study.

Narrow bands were designated every $1/24$ of the bandwidth, $B_i/24$, for each one-third octave band. The pressure spectrum level, also representing the energy over a bandwidth of 1 Hz, was computed at these discrete frequencies. These intermediate pressure spectrum levels, $L_S(f_{k,i})$, were obtained by linear interpolation between mid-band pressure spectrum levels $L_S(f_{m,i})$ of adjacent one-third octave-bands. The pressure spectrum levels above the highest-defined mid-band frequency were calculated by extrapolating from the two highest bands.

Contributions of the narrow bands surrounding the mid-band were added one-by-one to calculate an accumulated overall one-third octave-band level by the following process: Each pressure spectrum level $L_S(f_{k,i})$ including the mid-band level $L_S(f_{m,i})$, was: (1) adjusted for filter response, $A(f_{k,i})$, (2) converted to acoustic energy, (3) multiplied by $1/24$ the bandwidth of the corresponding one-third octave-band ($B_i/24$), and (4) summed on an energy basis to produce the calculated one-third octave-band level, $L_{CS}(f_i)$. The process of multiplying the energy-equivalent of each level by $B_i/24$ is analogous to integrating acoustic energy between subsequent values using a simple trapezoidal approximation.⁷

The process begins with the level at the exact mid-band frequency $f_{m,i}$, and continues alternately adding level data at the discrete frequency $kB_i/24$ lower than the mid-band frequency (negative k values) and then at the frequency $kB_i/24$ higher than the mid-band frequency (positive k values). The alternating process continues toward the lower and upper edge frequencies of the one-third octave-band i 's filter, decrementing and incrementing k by 1, until the calculated (i.e., reconstructed) one-third octave-band level, $L_{CS}(f_i)$, equals the value of the original as-measured sound pressure level, $L_{BS}(f_i)$:

$$L_{CS}(f_i) = 10 \log_{10} \left(\frac{B_i}{24} [10^{L'_S(f_{m,i})/10} + \sum_{k=-1}^{-24} 10^{L'_{LS}(f_{k,i})/10} + \sum_{k=1}^{24} 10^{L'_{US}(f_{k,i})/10}] \right) \quad [9]$$

where:

$$L'_S(f_{k,i}) = L_S(f_{k,i}) - A(f_{k,i});$$

$$A(f_{k,i}) = 10 \log_{10} \{1 + (4.5229)^6 (f_{k,i}/f_{m,i} - f_{m,i}/f_{k,i})^6\}, \quad [10]$$

= Butterworth⁶ filter attenuation;

$L'_{LS}(f_{k,i})$ = level adjusted for filter at frequencies lower than $f_{m,i}$, $k = -1$ to -24 ;

$L'_{US}(f_{k,i})$ = level adjusted for filter at frequencies higher than $f_{m,i}$, $k = +1$ to $+24$;

$f_{k,i} = f_{m,i} + kB_i/24$, discrete frequency within the one-third octave-band i 's filter, in Hz,
where, $k = 0$ for mid-band, and decremented by one for frequencies
lower than mid-band and incremented by one for frequencies higher
than mid-band;

$f_{1,i} = (10^{-1/20})f_{m,i}$, lower edge for base-10 design one-third octave-band filters;

$f_{2,i} = (10^{1/20})f_{m,i}$, upper edge for base-10 design one-third octave-band filters;

f_i = nominal mid-band frequencies for one-third octave-band, $i = 17$ to 40 corresponds to
bands from 50 Hz to 10 kHz; the range can be extended down to lower frequencies:
 $i = 14$ for the 25 Hz band, or up to higher frequencies: $i = 43$ for the 20 kHz band.

A second summation is simultaneously performed as above, steps (1) through (4). However, included in step (1) of this second summation is the adjustment for the attenuation effects of atmospheric absorption using the pure-tone algorithms, Equations [1] through [6], to obtain the adjusted narrow-band levels, $L''_S(f_{k,i})$. Note the absorption is additive for receiver-to-source distances (Equation [12], Case 2) and subtractive for source-to-receiver distances (Equation [13], Case 1). The resultant summation on an energy basis produces the calculated one-third octave-band level with both filter and atmospheric absorption effects, $L_{AS}(f_i)$:

$$L_{AS}(f_i) = 10 \log_{10} \left(\frac{B_i}{24} [10^{L''_S(f_{m,i})/10} + \sum_{k=-1}^{-24} 10^{L''_{LS}(f_{k,i})/10} + \sum_{k=1}^{24} 10^{L''_{US}(f_{k,i})/10}] \right) \quad [11]$$

where:

$$L''_S(f_{k,i}) = L_S(f_{k,i}) - A(f_{k,i}) + \alpha(f_{k,i})S \quad [12]$$

S = receiver-to-source distance in meters (Case 2);

$$L''_S(f_{k,i}) = L_S(f_{k,i}) - A(f_{k,i}) - \alpha(f_{k,i})S \quad [13]$$

S = source-to-receiver distance in meters (Case 1).

A base-10 designed third-order Butterworth⁶ filter shape was assumed. The adjustment for the attenuating effects, $A(f_{k,i})$, of a one-third octave-band filter at discrete frequencies, is based upon the well-known response equation for filters meeting the requirements of Type 1-X one-third octave-band Butterworth filter (Equation [10]). The one-third octave-band Butterworth filter is a common filter design in use in many of today's analyzers. It was also the filter response equation used by the authors of ANSI S1.26-1995 in the development of the Approximate Method³.

Typically, summation of the pressure spectrum levels for each one-third octave-band ends before the lower and upper filter band edge frequencies. Specifically, in this paper computation stopped when the calculated one-third octave-band level, $L_{CS}(f_i)$, was equal to the value of the original as-measured sound pressure level, $L_{BS}(f_i)$.

Finally, the difference between the calculated one-third octave-band level with filter effects, $L_{CS}(f_i)$, and the calculated one-third octave-band level with both filter and atmospheric absorption

effects, $L_{AS}(f_i)$, yielded the effective one-third octave-band attenuation by absorption, $\delta_{\text{eff}}(f_i)$ for the given temperature, humidity, pressure, and distance

$$\delta_{\text{eff}}(f_i) = L_{CS}(f_i) - L_{AS}(f_i) . \quad [14]$$

It was found that the process used in the Exact Method deviated from a true reciprocal process. In the receiver-to-source case the calculated absorption values diverge to unrealistically high absorption values versus those calculated for the source-to-receiver case under the same conditions over the same propagation distance (see Figure 3 and 4). Since absorption is additive in the receiver-to-source case, large corrected levels are computed. These are used in the energy summation process to extract the effective absorption. This is especially true at frequencies higher than mid-band in each one-third octave-band. The result is the high frequency portion of the one-third octave-band controls the absorption value calculated by dominating and overshadowing the contributions to the calculated value in the low frequency portion of the band. A less drastic process is evident in the source-to-receiver case where almost equal weighting is given to contributions of both the low and high frequency ends of the one-third octave-bands. For this reason, all comparisons made in this report versus the Exact Method are source-to-receiver comparisons.

2.2 Approximate Method

Annex E of the ANSI standard presents a simplified method for adapting the pure-tone algorithms to one-third octave-bands, known as the Approximate Method³. For the Approximate Method, the sound absorption for any one-third octave-band over the propagation path is equivalent to the product of: (1) the pure-tone sound attenuation rate, $\alpha(f_{m,i})$, computed using Equations [1] through [6] for the specified meteorological conditions at the exact mid-band frequency of the particular one-third octave-band filter; (2) the propagation path distance, s ; and (3) a theoretically-founded, experimentally-validated nonlinear function of the pure-tone attenuation and the normalized filter bandwidth, referred to herein as the “bandwidth adjustment function.” As shown in the standard, the sound absorption, $\delta_B(f_i)$, for any one-third octave-band, is computed as follows:

$$\delta_B(f_i) = [\alpha(f_{m,i})][s]\{1 + (B_r^2/10)[1 - (0.2303)[\alpha(f_{m,i})][s]]\}^{1.6} \quad [15]$$

where:

$\alpha(f_{m,i})$ = the sound attenuation rate computed at the exact mid-band frequency, f_m for one-third octave-band i , using Equations [1] through [6];

$f_{m,i}$ = the exact mid-band frequency, in Hz, as defined in Equation [8];

f_i = nominal mid-band frequencies for one-third octave-band filters; $i = 17$ to 40 corresponds to bands from 50 Hz to 10 kHz;

s = the propagation path distance in meters;

$B_r^2/10 = 0.0053254$, for base-10 designed one-third octave-band filters;

$B_r = (10^{1/20} - 10^{-1/20})$, for base-10 designed filters.

Note: The constant value of 0.2303 in Equation [15] is rounded from $2/[10 \log_{10}(e^2)]$.

The ANSI standard³ specifies that the above adaptation of pure-tone sound absorption provides an excellent measure of one-third octave-band absorption for the test spectra chosen in the development of the Approximate Method, assuming that the pure-tone attenuation over the total propagation path distance is less than 50 dB at the associated exact mid-band frequency.

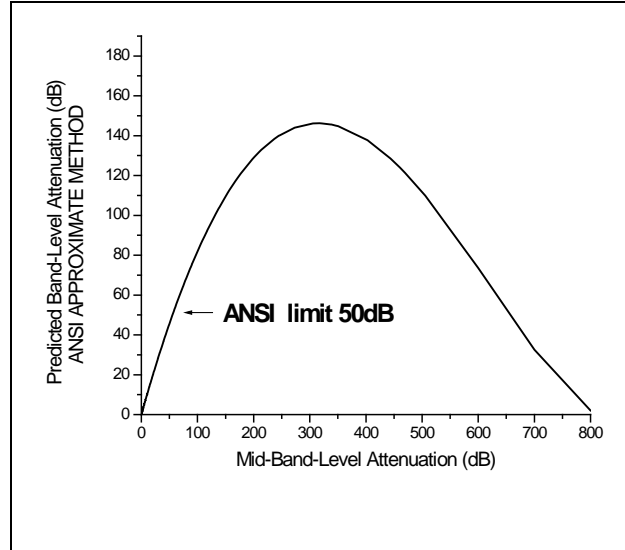


Figure 1: Approximate Method
ANSI Function – Approximate Method
Predicted vs. Mid-Band-Level Attenuation

The calculated attenuation (Equation [15]), plotted in Figure 1, is seen to increase from a minimum of 0 dB, with increasing mid-band attenuation up to a mid-band attenuation of approximately 300 dB, beyond which the calculated attenuation decreases with increasing mid-band attenuation. The attenuation calculated with Equation [15] was found to be in good agreement with the Exact Method up to the prescribed 50 dB ANSI limit³. Specifically, the Approximate Method root mean square error, when compared to the Exact Method over applicable frequency bands from 50 Hz to 10 kHz for altitudes up to 7620 m, was found to be less than 0.1 dB up to a mid-band level of 10 dB; increasing to 1.1 dB up to a mid-band level of 50 dB (see Table 4). Results were also measured relative to the Exact Method using the four spectrum shapes described in Section 3 and the eleven temperature and humidity (T/H) conditions described in Section 3.1. Above 50 dB (the ANSI limit³), the calculated attenuation diverges to unrealistically low values due to the limitations of the 2nd order equation.

The Approximate Method is a reciprocal process and is used for both Case 1 and Case 2 directions.

2.3 Proposed SAE Method

The proposed SAE Method is a simplified procedure for calculating attenuation by atmospheric

absorption on wideband sounds analyzed by one-third octave-band filters. It was developed using an approach similar to that used in the development of the Approximate Method. Specifically, the Exact Method presented in the ISO/ANSI^{2,3} standards was considered the reference by which the proposed SAE Method was judged. A base-10 designed third-order Butterworth⁶ filter shape was assumed. The same assumption was made by Joppa⁹ et al., and appears to be reasonable, since the traditional third-order Butterworth algorithms are used in common filter designs by many analyzer manufacturers.

The proposed SAE Method predicts band level attenuation by atmospheric absorption ($\delta_B(f_i)$ in decibels) using a two equation approach for mid-band attenuation levels less than 150 decibels, or greater than or equal to 150 decibels, as follows:

$$\delta_B(f_i) = A \times [\delta_t(f_{m,i})] \times (1 + B \times (C - D \times [\delta_t(f_{m,i})]))^E, \quad \delta_t(f_{m,i}) < 150 \text{ dB} \quad [16]$$

$$F + G \times [\delta_t(f_{m,i})], \quad \delta_t(f_{m,i}) \geq 150 \text{ dB} \quad [17]$$

where:

$\delta_t(f_{m,i}) = [\alpha(f_{m,i})][s]$ = mid-band (pure-tone) attenuation, in decibels;

$\alpha(f_{m,i})$ = mid-band attenuation coefficient, in decibels per meter, see Equation [1];

s = path length distance, in meters;

$A = 0.867942$;

$B = 0.111761$;

$C = 0.95824$;

$D = 0.008191$;

$E = 1.6$;

$F = 9.2$;

$G = 0.765$;

f_i = nominal mid-band frequencies for one-third octave-band, $i = 17$ to 40 corresponds to bands from 50 Hz to 10 kHz;

$i = 17, \dots, 40$, integers represent one-third octave-band from 50 Hz to 10 kHz; the range can be extended down to lower frequencies: $i = 14$ for the 25 Hz band, or up to higher frequencies: $i = 43$ for the 20 kHz band; and

$f_{m,i}$ = the exact mid-band frequency, in Hz, as defined in Equation [8].

The proposed SAE Method is a reciprocal process and can be used for both source-to-receiver and receiver-to-source directions (Case 1 and Case 2).

Frequency-distance pairs that yield large attenuations, such as those over 150 dB, are unlikely to have a significant impact on audible broadband levels. However, curve-fit generated methods that limit their scope only to accuracy at the smaller attenuation levels can exhibit irregular behavior for larger mid-band attenuations. The Approximate Method, for example, follows a direct relationship between predicted band-level attenuation and mid-band-level attenuation only up to mid-band attenuations of approximately 300 dB. After this point, the relationship is inverted, as shown in Figure 1. This kind of

breakdown of the model can introduce errors when mid-band attenuation levels are large. Therefore, the goal in extending the valid attenuation range is to ensure the correct trend of attenuation is maintained and anomalies are avoided.

2.4 SAE 866A Method

The SAE 866A Method has served, to date, as the basis for many analyses of noise propagation and for correcting sound propagation levels measured under given atmospheric conditions to specified reference conditions. Based on the theory of Kneser¹⁰, it is a pure-tone method where the sound absorption for any one-third octave band over the propagation path for the specified meteorological conditions is equivalent to the product of: (1) the sound attenuation rate, and (2) the propagation path distances. The sound absorption rate is computed as described in the Society of Automotive Engineers Aerospace Recommended Practice 866A, “Standard Values of Atmospheric Absorption as a Function of Temperature and Humidity” (SAE ARP 866A)⁴, at (a) the exact mid-band frequency of the particular one-third octave-band filter up to and including 4000 Hz; and (b) the SAE 866A lower-band edge-frequencies for mid-band frequencies greater than 4000 Hz as follows:

$$\delta_B(f_i) = [\alpha(f_{m,i})][s], \quad i = 17 \text{ to } 36 \quad [18]$$

$$[\alpha(f_{\text{edge},i})][s], \quad i = 37 \text{ to } 40 \quad [19]$$

where:

$\alpha(f_{m,i})$ = the sound attenuation rate computed at the preferred mid-band frequency for each one-third octave-band up to and including 4000 Hz using the equations of Reference 4;

$\alpha(f_{\text{edge},i})$ = the sound attenuation rate computed at edge-frequencies equal to 4500, 5600, 7100, and 9000 Hz for bands greater than 4000 Hz using the equations found in Reference 4;

f_i = nominal mid-band frequencies for one-third octave-band, $i = 17$ to 40 corresponds to bands from 50 Hz to 10 kHz;

$f_{m,i}$ = the exact mid-band frequency, in Hz, as defined in Equation [8];

$f_{\text{edge},i}$ = 4500, 5600, 7100, and 9000 Hz for $i = 37, 38, 39$, and 40, respectively; and

s = the propagation path distance.

The SAE 866A Method is a reciprocal process and is used for both source-to-receiver and receiver-to-source propagation distances (Case 1, Case 2, and Case 3). Readers are referred to Reference 4 for further discussion of the SAE 866A methodology.

3. DEVELOPMENT OF THE PROPOSED SAE METHOD

Spectral analysis of wideband sounds using one-third octave-band filters yields sound pressure levels in one-third octave frequency bands. This analysis is a requirement of Part 36¹ for aircraft noise certification. The sound pressure levels include the effects of filter attenuation response characteristics as well as the attenuation introduced by atmospheric absorption. The magnitude of these effects was found to vary with the slope of the test spectrum.

The proposed SAE Method was developed using a set of shaped broadband (non-tonal) spectra chosen to represent a wide range of spectrum slopes that may be encountered in typical aircraft data. These included four non-tonal test spectra with data slopes of +5, 0, -2, and -5 dB per one-third octave-band, shown in Figure 2. Atmospheric absorption was calculated with the Exact Method to fixed receiver distances for these shaped source spectra under different atmospheric conditions, described in Section 3.1. The result was a set of points relating one-third octave-band attenuation by atmospheric absorption, with mid-band-level attenuation for each one-third octave-band.

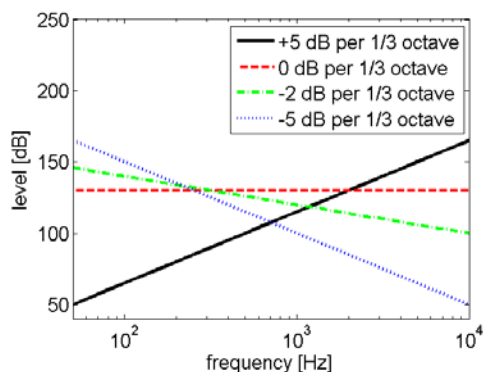


Figure 2: Non-Tonal Test Spectra
Data Slopes: -5, -2, 0, +5 dB per 1/3 octave-band

A commercial statistical regression program was used on these data points to develop an empirical equation relating representative one-third octave-band atmospheric attenuation with mid-band attenuation for a particular one-third octave band. The equations were evaluated by comparing their relative goodness-of-fit, using the R^2 coefficient of determination. The general form of the equation for the ANSI S1.26-1995 Approximate Method was used as a starting point because it was found to be in good agreement with the Exact Method for small values of absorption. The goal of the analysis, using the data derived from the Exact Method, was to increase the accuracy and extend the applicability of the Approximate Method to well beyond the limited 50 dB path-length absorption.

3.1 Temperature / Humidity / Static Pressure

From a grid of 22 temperature and humidity (T/H) points covering the Part 36 temperature and humidity window, a more manageable set of eleven fixed temperature and humidity points were selected. The points were selected as representative of the attenuation characteristics within the window after comparing the band level versus mid-band level attenuation for all 22 points. The selected points were at the extremes and at the center of the Part 36 accepted temperature and humidity window and include the Part 36 reference temperature and humidity point (25°C / 70%RH) and the temperature and humidity point used by Joppa⁹ et al. in the development of the Approximate Method (6°C / 35%RH).

The eleven selected temperature and humidity points are presented in Table 1.

Table 1. Selected Temperature and Humidity Points

Temperature (°C)	Relative Humidity (%RH)
25	70
6	35
6	49
6	95
10	42
15	60
15	95
21	27
21	95
32	20
32	95

For the purpose of the development of this procedure, the atmosphere was assumed to be homogeneous and uniform with constant atmospheric pressure at the ISA sea level value of 101.325 kPa.

3.2 Data Processing

A computer program was developed to process data and to calculate attenuation by atmospheric absorption with the Exact Method as described in Section 2.1 above. It was subsequently augmented by a macro-driven routine written for an EXCEL spreadsheet. Each of the sloped test spectra was processed at each of the eleven temperature and humidity data points.

The receiver-to-source distances were chosen to produce attenuation values in the 200 to 500 dB ranges in the upper frequency bands. From plots of the data, it was determined that data processed at 25°C / 70%RH produced data curves that reasonably approximated data from the other ten temperature and humidity points. The 25°C / 70%RH one-third octave-band attenuation versus mid-band attenuation is shown plotted in Figure 3 and 4 for the four shaped test spectra for both the source-to-receiver and receiver-to-source cases (Case 1 and Case 2).

Note that in Figure 4, the receiver-to-source case (Case 2), the calculated absorption values diverge to unrealistically high absorption values as compared with those calculated for the source-to-receiver case (Case 1), shown in Figure 3, under the same conditions over the same propagation distance. For this reason, all comparisons made in this report versus the Exact Method are Case 1, source-to-receiver comparisons (See Section 2.1).

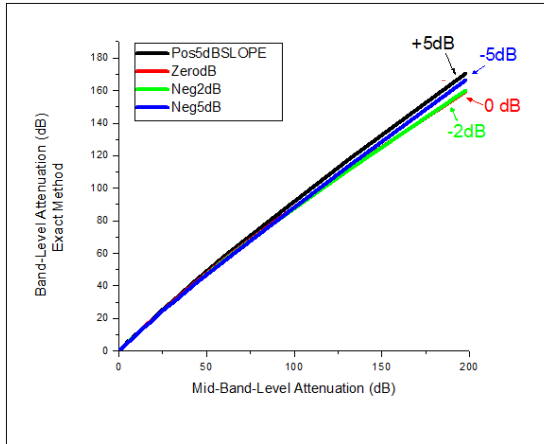


Figure 3: Case 1, Source-to-Receiver
Data Slopes: +5, 0, -2, -5 dB per 1/3 octave
25°C, 70% RH, Pressure 101.325 kPa

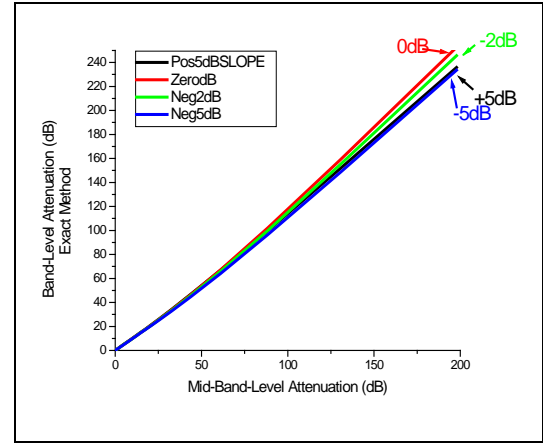


Figure 4: Case 2, Receiver-to-Source
Data Slopes: +5, 0, -2, -5 dB per 1/3 octave
25°C, 70% RH, Pressure 101.325 kPa

3.3 Algorithm Generation

Least-squares regression techniques were applied to Case 1 atmospheric absorption data versus mid-band attenuation data obtained from processing the four shaped spectra at 25°C / 70%RH (see Figure 3) with a goal of developing an appropriate algorithm. The equations were evaluated by comparing their relative goodness-of-fit, using the R^2 coefficient of determination.

Because the equation for the ANSI S1.26-1995 Approximate Method was found to be in good agreement with the Exact Method for lower attenuation levels, its general form was used as a starting point in the regression analysis with a variable initially introduced for data slope. Preliminary results and Figure 3 show that the complexity of an equation with slope as an added variable would not provide a worthwhile improvement in accuracy over an equation with a fixed slope. A final regression equation was thus normalized with a fixed slope of 3 dB per one-third octave-band for mid-band levels less than 150 dB and is shown in Section 2.3, Equation [16]. The test spectra were further processed at large propagation distances (producing large values of atmospheric attenuation) to extend the usefulness of the proposed SAE Method for mid-band attenuation levels ≥ 150 dB. This yielded a second regression equation (Equation [17]) tangent to the first (Equation [16]) at the 150 dB point. The discontinuity at 150 dB is less than 0.05 dB.

3.4 Error Analysis

The algorithms (equations [16], [17]) developed for the proposed SAE Method were used to predict atmospheric absorption for each of the four-sloped spectra at each of the eleven temperature and humidity data points of Section 3.1. The predicted attenuation values were compared against data obtained using the Exact Method assuming a homogeneous atmosphere and a static pressure of 101.325 kPa for each of the eleven temperature and humidity data points.

Error curves (Proposed SAE Method minus Exact Method) are shown in Appendix A, Figures A1 to A4. In each figure, curves are plotted for the eleven temperature and humidity points across mid-band attenuations of up to 1000 dB. Figures A1a, A2a, A3a, and A4a show results for the 0, -2, +5, and -5 dB spectral data slopes, respectively. Figures A1b to A4b zoom in on the first 60 dB of mid-band

attenuation data from Figures A1a to A4a. The error in the predicted data from these curves is summarized in Table 2.

Table 2. One-Third Octave-Band Error Data Predicted minus Exact Levels (4 Data Slopes)

Mid-Band Attenuation (dB)	Error (Predicted Level)	
	10 T/H Points (dB)	(32°C / 95%RH) (dB)
0-10	≤0.5	≤0.5
10-30	≤1.0	≤1.0
30-60	≤3.5	≤3.5
60-100	≤10	≤15
100-150	≤15	≤25
150-300	≤20	≤35
300-500	≤30	≤35
500- 700	≤60	no data

As seen in Table 2, the error in the predicted attenuation using the proposed SAE Method for ten of the temperature and humidity data points is less than 10% of the mid-band attenuation. The 32°C / 95% RH data point at the high temperature/high humidity extreme of the Part 36 testing window is the exception with errors up to 20% of the mid-band attenuation.

4. SENSITIVITY TESTING

A measure of the sensitivity of the proposed SAE Method to aircraft data was obtained using an approximation to a true spectrum shape. Figure 5 depicts an approximation, at the source, of a one-third octave-band spectrum shape for typical commercial jet aircraft at high engine power settings. This test spectrum, used by Joppa⁹ et al. in the development of the Approximate Method, was used herein. Six conditions were tested as follows (comparisons are between the proposed SAE and Exact Methods, unless otherwise noted):

1. (Section 4.1), Static atmospheric pressure of 101.325 kPa;
2. (Section 4.2), Lapsed pressure changes with altitude;
3. (Section 4.3), Lapsed pressure changes using 30 meter altitude layers;
4. (Section 4.4), Changing filter shapes;
5. (Section 4.5), Comparison versus the SAE 866A Method; and
6. (Section 4.6), Extending the applicable frequency range to 20 kHz.

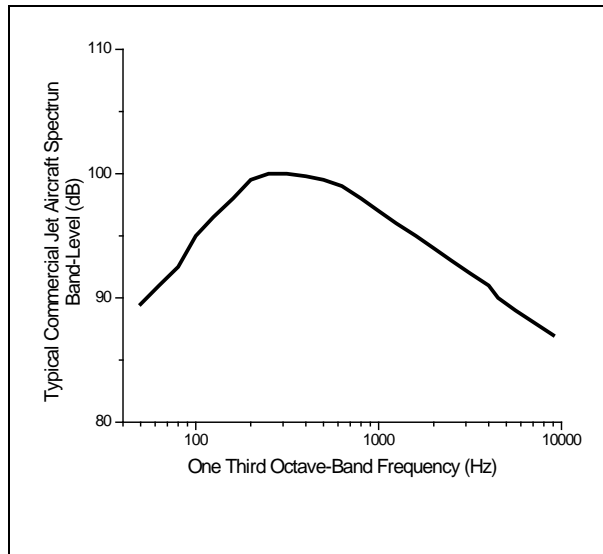


Figure 5: Commercial Jet Aircraft

Typical One-Third Octave-Band Noise Levels at 1 m from the Source at High Engine Power Setting⁹

Comparisons for the six conditions were performed at eight propagation distances from 75 to 7620 meters at the following four temperature and humidity points at the extremes and center of the Part 36 temperature and humidity window:

32°C / 20 %RH,
32°C / 95 %RH,
6 °C / 49 %RH,
25°C / 70 %RH.

For each condition, the source one-third octave-band noise data was corrected for the effects of

atmospheric absorption using the proposed SAE, Exact, and/or SAE 866A Methods. The attenuated one-third octave-band data (50 Hz to 10 kHz) were further used to compute the Perceived Noise Level denoted by the symbol L_{PN} , and the A-weighted Noise Level denoted by the symbol L_A . Level-difference data ΔL_A and ΔL_{PN} for the four conditions comparing the proposed SAE and Exact Methods (calculated as proposed SAE Method minus Exact Method), at the four temperature and humidity points and across the eight altitudes (75 to 7620 meters), are presented in Appendix B, Tables B1-B4. Level difference data for the condition comparing the SAE 866A and proposed SAE Methods (calculated as SAE 866A Method minus proposed SAE Method) are presented in Appendix B, Table B5.

Level-difference curves versus altitude for the calculated adjusted levels (Case 1, source-to-receiver) for 24 one-third octave-bands are presented in Appendix C, Figures C1a-d through C6a-d. The mid-band attenuation at 7620 meters for the 10 kHz one-third octave-band (band 40) is provided above each figure as a reference point, except for the conditions of Section 4.6, which include the mid-band attenuation at 2400 meters of bands 40 and 43.

The L_A and L_{PN} level-difference data tabulations obtained at the four temperature and humidity points were combined and are presented in Figure 6 and 7 for the first four conditions tested. Included for both metrics are the mean, maximum, minimum and $\pm 2\sigma$ range of values (± 2 standard deviations) combined over altitudes from 75 m to 1200 m. This altitude range was chosen to provide an equivalent comparison between the A-weighted and Perceived Noise metrics; while L_A can be calculated for all eight altitudes, only the five lower altitudes have a sufficient number of one-third octave-bands that fall within the acceptable level range to calculate L_{PN} . L_A statistics across the full altitude range, to 7620 m, are provided in the accompanying text.

Level-difference data obtained for the four temperature and humidity points were combined and are provided along with the combined mid-band attenuation levels versus altitude in Figures 8 to 19 for the six conditions tested. Included are the mean, maximum, minimum and $\pm 2\sigma$ range of values (± 2 standard deviations).

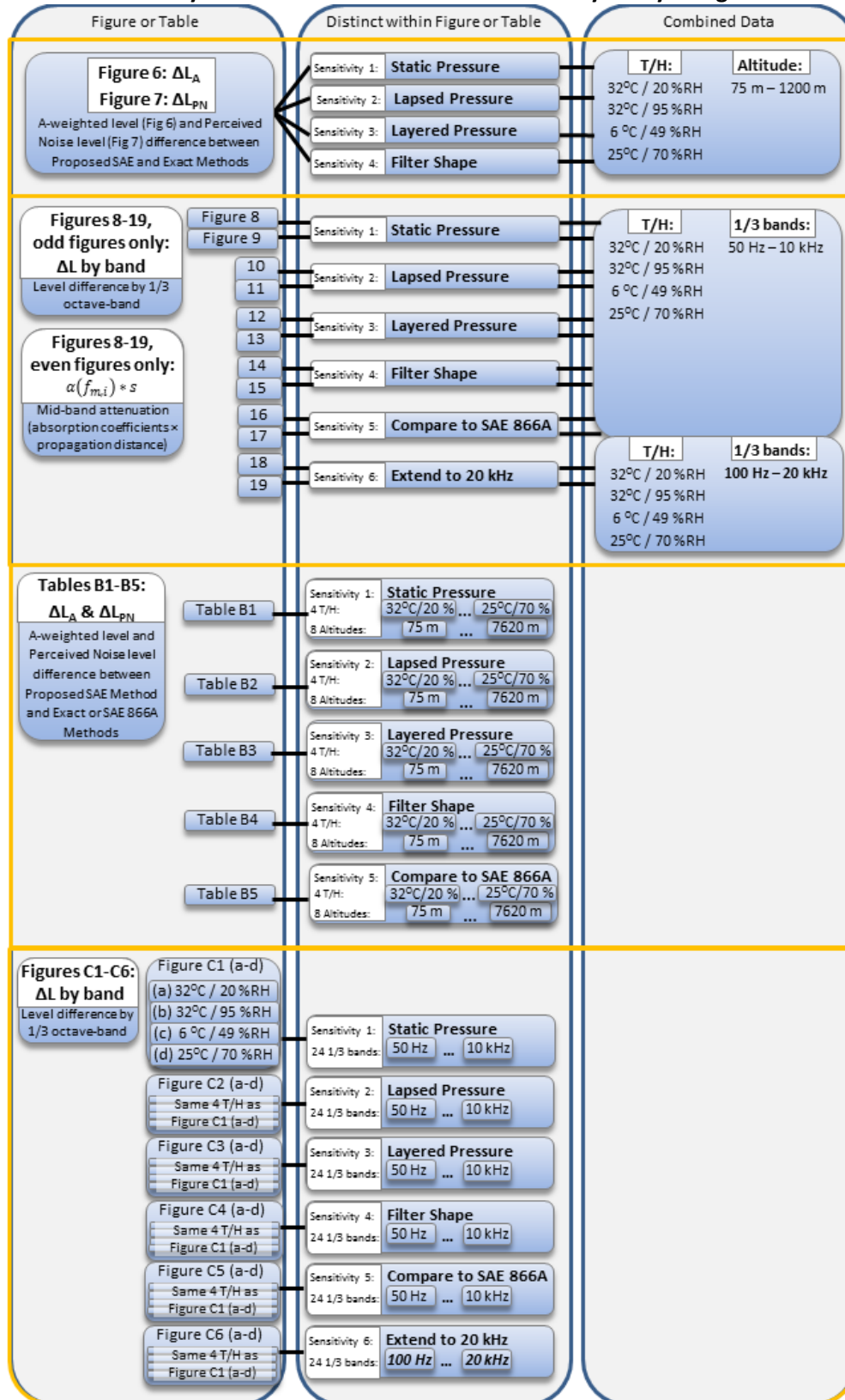
Table 3 shows a summary of the different ways the data are organized and rendered in this report, including the metric of the data presented, categorization of the conditions for which the data is grouped per figure or table, and distinctions and/or groupings made of the data within each figure.

4.1 Homogeneous Atmosphere with Constant Pressure (101.325 kPa)

For these comparisons, a homogenous atmosphere was assumed, i.e. temperature, humidity and pressure did not vary with altitude. Atmospheric pressure was set to a static, ISA sea level value of 101.325 kPa.

Atmospheric absorption adjustments were made to the source one-third octave-band noise data at eight altitudes from 75 to 7620 meters at the four temperature and humidity conditions selected. Presented in Appendix B, Table B1 are the source-to-receiver level-difference data obtained comparing the proposed SAE Method with the Exact Method for the A-weighted and Perceived Noise metrics (ΔL_A and ΔL_{PN}).

Table 3. Summary of the Data Presentation in Sensitivity Analysis Figures and Tables



Note, in Figures 6 and 7, which show the L_A and L_{PN} level-difference data combined over the four temperature and humidity points over the altitudes up to 1200 meters, the mean differences for ΔL_A and ΔL_{PN} are -0.04 dB and -0.06 dB, respectively, and both have 2σ values of ± 0.05 dB. For altitudes up to 7620 m, the mean and 2σ statistics of ΔL_A are -0.07 dB and ± 0.08 dB, respectively.

Presented in Appendix C, Figure C1 (a-d) are level-difference curves (proposed SAE Method minus the Exact Method) for the calculated one-third octave-band source-to-receiver adjustment levels (50 Hz to 10 kHz) for each of the four selected temperature and humidity points. The mid-band attenuation level at 7620 meters for the 10 kHz band (band 40) is included at the top of each figure as a point of reference.

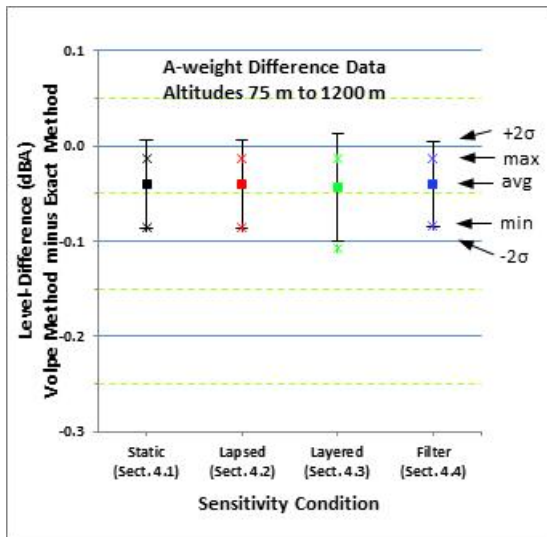


Figure 6: ΔL_A - Level-Difference Data

Combined Data – 4 T/H Points
Five Altitudes, 75 to 1200 meters
Method Comparisons for Four Sets of Conditions
see Sections 4.1, 4.2, 4.3, 4.4

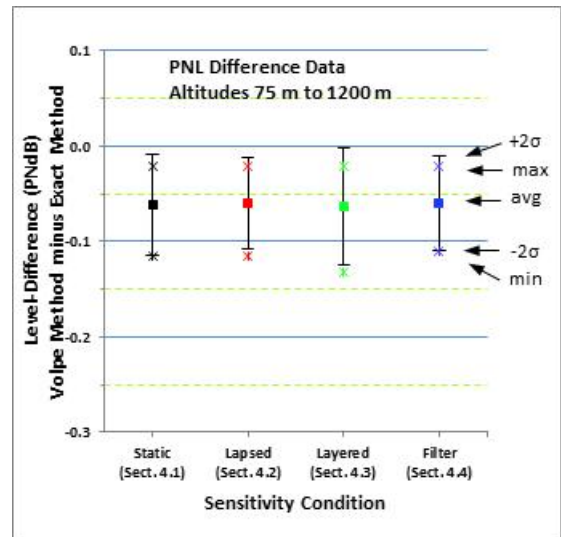


Figure 7: ΔL_{PN} - Level-Difference Data

Combined Data – 4 T/H Points
Five Altitudes, 75 to 1200 meters
Method Comparisons for Four Sets of Conditions
see Sections 4.1, 4.2, 4.3, 4.4

The one-third octave-band level-difference data of Figure C1 (a-d), for the four temperature and humidity points, were combined and are presented in Figure 8. Shown in Figure 8 is the level-difference as a function of altitude ranging from 75 to 7620 meters. Mid-band attenuation level data are presented in Figure 9 as a function of altitude. The mean, maximum, minimum, and $\pm 2\sigma$ values (± 2 standard deviations) calculated across the 24 one-third octave-bands are included.

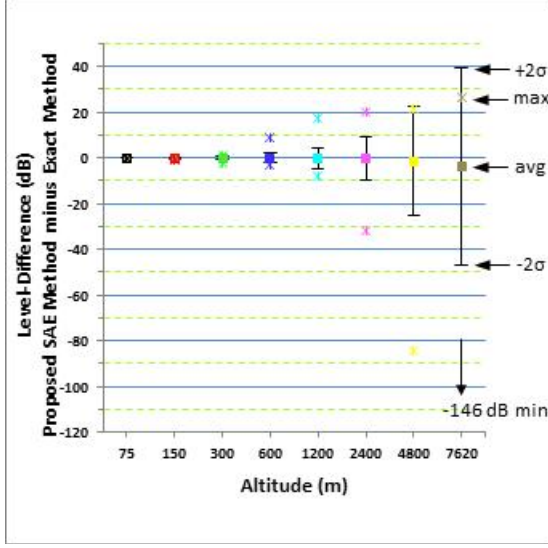


Figure 8: One-Third Octave-Band
Level-Difference Data
Combined Data – 4 T/H Points
Proposed SAE Method minus Exact Method
Static Pressure 101.325 kPa

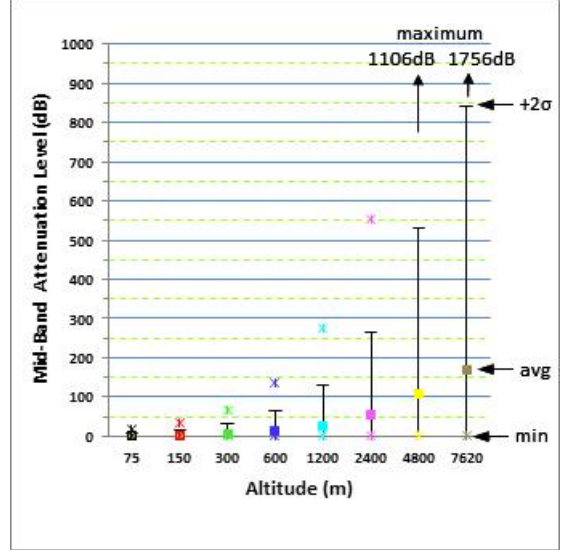


Figure 9: Mid-Band Level Data
Combined Data – 4 T/H Points
Static Pressure 101.325 kPa

Statistics for the sensitivity conditions are summarized in Table 4. Differences in comparisons between methods are presented as the root mean square error (RMSE), which accounts for both the variation of the data of the investigated method, as well as its bias from the results of the method to which it is compared. The RMSE is presented as a function of mid-band attenuation level, combining the data across one-third octave-bands and altitudes whose mid-band attenuations fit within each specified range. RMSE is calculated as

$$RMSE = \sqrt{(1/N) \sum_{i=1}^N (\Delta L_i)^2} \quad [20]$$

where ΔL_i is the difference between the investigated method and the method to which it is compared (in the first four sensitivity conditions, the proposed SAE Method is compared to the Exact Method), and N is the number of data points with mid-band attenuation levels in a given range. RMSE results for the static pressure condition of 101.325 kPa are presented in the first results column of Table 4. Note in Table 4 that, through 50 dB of mid-band attenuation, an error of 0.8 dB was calculated with the proposed SAE Method, which is less than the 1.1 dB error for the Approximate Method.

In addition, the useable range of the proposed SAE Method is seen to extend to over 500 dB of mid-band attenuation with RMSE of less than 0.1 dB to 10 dB and 3% (of mid-band attenuation) to 100 dB and 500 dB.

Similar results were obtained (not shown) extending the frequency range of the proposed SAE Method down to the 25Hz one-third octave-band. This frequency range would be applicable for helicopter noise data processing.

Table 4. Root Mean Square Error Statistics for One-Third Octave-Band Data
Level-Difference versus Mid-Band Attenuation

Proposed SAE Method minus Exact Method
75 to 7620 meters

Mid-Band Attenuation dB	Static Pressure (Section 4.1) dB	Lapsed Pressure (Section 4.2) dB	30m Layers w/ Lapsed Pressure (Section 4.3) dB	LD _{long} Filter Static Pressure (Section 4.4) dB	SAE 866A Method ^A Static Pressure (Section 4.5) dB	20 kHz Static Pressure (Section 4.6) dB	Approx. Method ^B Static Pressure (Section 4.1) dB
0-10	0.1	0.1	0.1	0.1	1.1	0.1	0.0
10-20	0.3	0.3	0.3	0.3	2.7	0.3	0.1
20-50	0.8	0.8	0.8	0.6	4.9	0.8	1.1
50-100	2.5	2.3	1.6	2.4	12.0	2.4	N/A ^C
100-200	8.8	7.2	3.1	10.7	26.8	8.3	N/A ^C
200-500	14.1	14.1	7.3	18.2	40.8	18.4	N/A ^C
500-700	24.0	27.4	13.6	30.2	49.7	28.1	N/A ^C

^A - SAE ARP 866A Method minus Proposed SAE Method
^B - Approximate Method minus Exact Method
^C - Approximate Method is not valid for mid-band attenuations larger than 50 dB

4.2 Homogeneous Atmosphere with Lapsed Pressure

For these comparisons, a homogenous atmosphere was assumed, i.e., temperature and humidity did not vary with altitude. However, atmospheric pressure was taken into account to an altitude of 7620 meters using the standard ISO pressure lapse rate defined by Equation [21]. The atmospheric pressure calculated at the source altitude was assumed constant over the full propagation path.

The standard ISO pressure lapsed rate in kPa was calculated as follows:

$$\text{Pressure} = 101.325 \times 10^{(-5.256 \times 10^{-5} \times \text{alt})} \quad [21]$$

where:

alt = altitude above mean sea level, in meters

Atmospheric absorption adjustments were made to the source one-third octave-band noise data at eight altitudes from 75 to 7620 meters at the four temperature and humidity conditions selected. Presented in Appendix B, Table B2 are the source-to-receiver level-difference data obtained comparing the proposed SAE Method with the Exact Method for the A-weighted and Perceived Noise metrics (ΔL_A and ΔL_{PN}).

As in Section 4.1, the level-difference data of Table B2 (proposed SAE Method minus the Exact Method) for the ΔL_A and ΔL_{PN} metrics obtained at the four temperature and humidity conditions were combined and are presented in Figures 6 and 7. Included are the mean, maximum, minimum and $\pm 2\sigma$ values (± 2 standard deviations) over the four temperature and humidity conditions and the five altitudes up to 1200 m. Note in Figures 6 and 7 that over this altitude range, the mean difference for ΔL_A and ΔL_{PN} are -0.04 dB and -0.06 dB, respectively, and both have 2σ values ± 0.05 dB. For altitudes up to 7620 m, the mean and 2σ statistics of ΔL_A are -0.07 dB and ± 0.08 dB, respectively.

Presented in Appendix C, Figure C2 (a-d) are level-difference curves (proposed SAE Method minus the Exact Method) for the calculated one-third octave-band source-to-receiver atmospheric absorption adjustment levels (50 Hz to 10 kHz) for each of the four selected temperature and humidity points. The

mid-band attenuation level at 7620 meters for the 10 kHz band (band 40) is included at the top of each figure as a point of reference.

The one-third octave-band level-difference data of Figure C2 (a-d), for the four temperature and humidity points, were combined and are presented in Figure 10. Shown in Figure 10 is the level-difference over the altitude range of 75 to 7620 meters. Mid-band level data are presented in Figure 11. Included are the mean, maximum, minimum, and $\pm 2\sigma$ values (± 2 standard deviations) calculated across the 24 one-third octave-bands.

RMSE statistics for the one-third octave-band level-difference data of Figures 10 and 11 are included in the second results column of Table 4. Note that introducing lapsed pressure resulted in little difference in computed levels when compared with the static pressure conditions of Section 4.1.

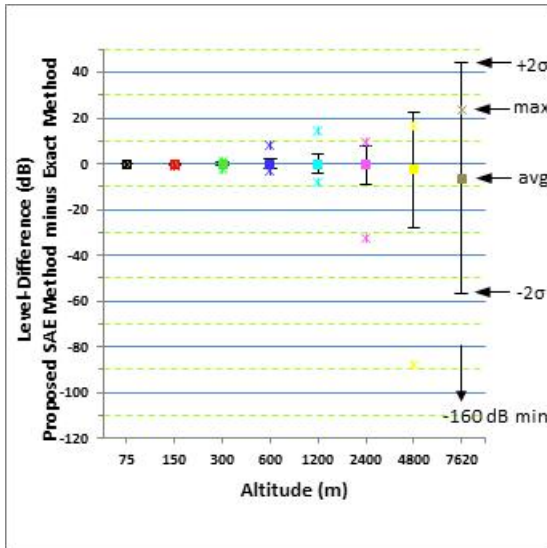


Figure 10: One-Third Octave-Band
Level-Difference Data
Combined Data – 4 T/H Points
Proposed SAE Method minus Exact Method
w/ Lapsed Pressure at Altitude

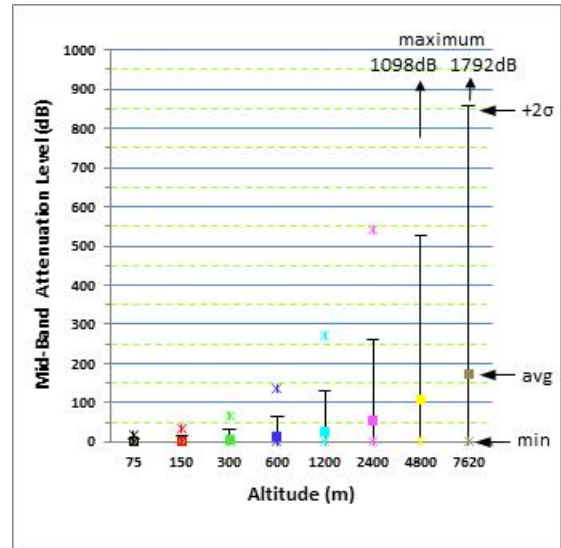


Figure 11: Mid-Band Level Data
Combined Data – 4 T/H Points
w/ Lapsed Pressure at Altitude

4.3 30-meter Layers with Lapsed Pressure

For these comparisons, the atmosphere was divided into equal altitude layers of 30-meters each. A homogenous atmosphere was assumed, i.e., temperature and humidity did not vary with altitude.

Atmospheric pressure changes were taken into account to an altitude of 7620 meters using Equation [21]. Lapsed pressure was calculated at the center of each layer at the appropriate altitude. That pressure was used to represent the pressure for that layer in the calculations for both the Exact and proposed SAE Methods for the four temperature and humidity conditions tested over the altitude range of 75 to 7620 meters.

Atmospheric absorption adjustments were made to the source one-third octave-band noise data at eight altitudes from 75 to 7620 meters at the four temperature and humidity conditions selected. Presented in Appendix B, Table B3 are the source-to-receiver level-difference data obtained comparing the proposed SAE Method with the Exact Method for the A-weighted and Perceived Noise metrics (ΔL_A and ΔL_{PN}).

The level-difference data of Table B3 (proposed SAE Method minus the Exact Method) for the ΔL_A and ΔL_{PN} metrics obtained at the four temperature and humidity conditions were combined and are presented in Figures 6 and 7. Included are the mean, maximum, minimum and $\pm 2\sigma$ values (± 2 standard deviations) over the four temperature and humidity conditions and the five altitudes up to 1200 m. Note in Figures 6 and 7 that over this altitude range, the mean difference for ΔL_A and ΔL_{PN} are -0.04 dB and -0.06 dB, respectively, and both have 2σ values of ± 0.06 dB. For altitudes up to 7620 m, the mean and 2σ statistics of ΔL_A are -0.08 dB and ± 0.11 dB, respectively.

Presented in Appendix C, Figure C3 (a-d) are level-difference curves (proposed SAE Method minus the Exact Method) for the calculated one-third octave-band source-to-receiver adjustment levels (50 Hz to 10 kHz) for each of the four selected temperature and humidity points. The mid-band attenuation level at 7620 meters for the 10 kHz band (band 40) is included at the top of each figure as a point of reference.

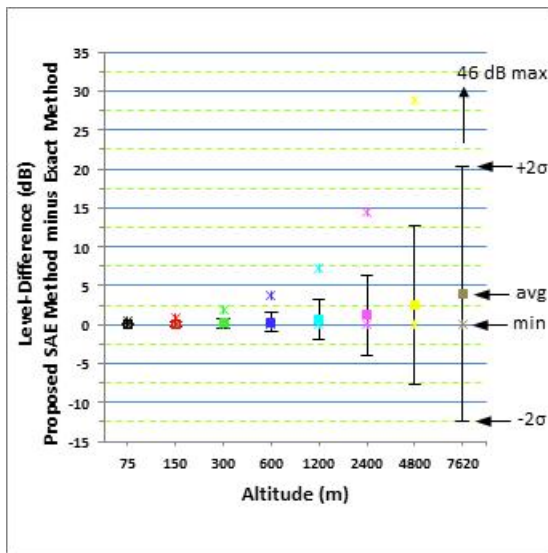


Figure 12: One-Third Octave-Band
Level-Difference Data
Combined Data – 4 T/H Points
Proposed SAE Method minus Exact Method
30-meter Layers w/Pressure at Altitude

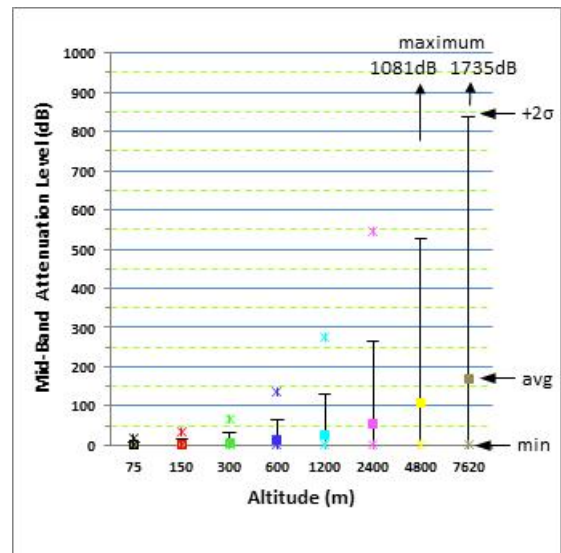


Figure 13: Mid-Band Level Data
Combined Data – 4 T/H Points
30-meter Layers w/Pressure at Altitude

The one-third octave-band level-difference data of Figure C3 (a-d), for the four temperature and humidity points, were combined and are presented in Figure 12. Shown in Figure 12 is the level-difference over the altitude range of 75 to 7620 meters. Mid-band level data is presented in Figure 13. Included are the mean, maximum, minimum and $\pm 2\sigma$ values (± 2 standard deviations) calculated across the 24 one-third octave bands.

RMSE statistics for the one-third octave-band level-difference data of Figures 12 and 13 are included in the third results column of Table 4. Note the level-difference is improved when compared to the results of the static pressure condition (Section 4.1) and the lapsed pressure condition (section 4.2). This is to be expected since the proposed SAE Method is repeatedly used in each 30-meter altitude layer in the region of its greatest accuracy, i.e. in the mid-band attenuation range of 0 to 50 dB.

4.4 Filter Shape

To obtain a measure of the sensitivity of the proposed SAE Method to one-third octave filter shapes, the base-10 designed third-order Butterworth⁶ filter shape (used in the Exact Method, as described in the above sections) was modified to more closely approximate an “ideal” filter, i.e., a filter with infinite attenuation characteristics outside of the pass-band.

Using the data of Reference 11, an equation was developed to simulate the attenuation characteristics of the “long” filter shape found in the Larson Davis Laboratories, (LD) Model 2900 analyzer. The so-called “long” filter shape (known herein as LD_{Long}) represents the manufacturers attempt to simulate an “ideal” filter. The equation developed is as follows:

$$A(f_{k,i}) = 90 \log_{10}\{1 + (2.8071)^6(f_{k,i}/f_{m,i} - f_{m,i}/f_{k,i})^6\}, \quad [22]$$

where:

$$f_{k,i} = f_{m,i} + kB_i/24.$$

In this section, Equation [22] was substituted for Equation [10] for the Exact (LD_{Long}) Method processing. Also, a homogenous atmosphere was assumed, i.e. temperature, humidity and pressure did not vary with altitude. Atmospheric pressure was set to a static, ISA sea level value of 101.325 kPa.

Atmospheric absorption adjustments were made to the source one-third octave-band noise data at eight altitudes from 75 to 7620 meters at the four temperature and humidity conditions selected. Presented in Appendix B, Table B4 are the source-to-receiver level-difference data obtained comparing the proposed SAE Method with the Exact (LD_{Long}) Method for the A-weighted and Perceived Noise metrics (ΔL_A and ΔL_{PN}).

The level-difference data of Table B4 (proposed SAE Method minus the Exact (LD_{Long}) Method) for the ΔL_A and ΔL_{PN} metrics obtained at the four temperature and humidity conditions were combined and are presented in Figures 6 and 7. Included are the mean, maximum, minimum and $\pm 2\sigma$ values (± 2 standard deviations) over the four temperature and humidity conditions and the five altitudes up to 1200 m. Note in Figures 6 and 7 that over this altitude range, the mean difference for ΔL_A and ΔL_{PN} are -0.04 dB and -0.06 dB, respectively, and both have 2σ values of ± 0.05 dB. For altitudes up to 7620 m, the mean and 2σ statistics of ΔL_A are -0.06 dB and ± 0.07 dB, respectively.

Presented in Appendix C, Figure C4 (a-d) are level-difference curves (proposed SAE Method minus the Exact Method) for the calculated one-third octave-band source-to-receiver adjustment levels (50 Hz to 10 kHz) for each of the four selected temperature and humidity points. The mid-band attenuation level at 7620 meters for the 10 kHz band (band 40) is included at the top of each figure as a point of reference.

The one-third octave-band level-difference data of Figure C4 (a-d), for the four temperature and humidity points, were combined and are presented in Figure 14. Shown in Figure 14 is the level-difference over the altitude range of 75 to 7620 meters. Mid-band level data is presented in Figure 15. Included are the mean, maximum, minimum, and $\pm 2\sigma$ values (± 2 standard deviations) calculated across the 24 one-third octave-bands.

RMSE statistics for the one-third octave-band level-difference data of Figures 14 and 15 are included in the fourth results column of Table 4. Note that introducing a drastic change in the filter shape increases the level-difference errors by less than 1% of the mid-band attenuations when compared with the results obtained in Section 4.1 using the third-order Butterworth filter shape.

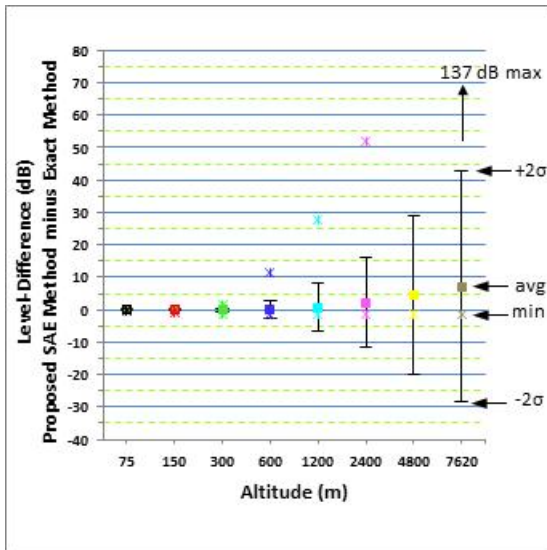


Figure 14: One-Third Octave-Band
Level-Difference Data
Combined Data – 4 T/H Points
Proposed SAE Method minus
Exact (LD_{Long}) Method
Static Pressure 101.325 kPa

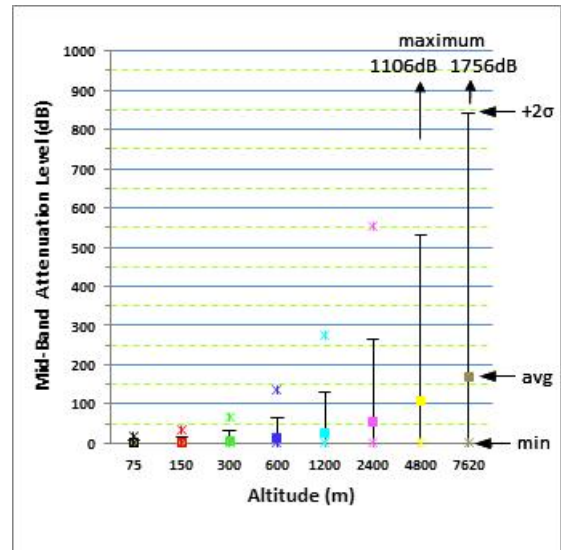


Figure 15: Mid-Band Level Data
Combined Data – 4 T/H Points
Static Pressure 101.325 kPa

4.5 SAE 866A Method

The proposed SAE Method was compared with the SAE 866A Method as described in the Society of Automotive Engineers Aerospace Recommended Practice 866A, “Standard Values of Atmospheric Absorption as a Function of Temperature and Humidity” (SAE ARP 866A)⁴.

For these comparisons, a homogenous atmosphere was assumed, i.e. temperature, humidity and pressure did not vary with altitude. For the proposed SAE Method calculations, atmospheric pressure was set to a static, ISA sea level value of 101.325 kPa for the four temperature and humidity conditions (pressure is not a variable in the SAE 866A procedure).

Atmospheric absorption adjustments were made to the source one-third octave-band noise data at eight altitudes from 75 to 7620 meters at the four temperature and humidity conditions selected. Presented in Appendix B, Table B5 are the source-to-receiver level-difference data obtained comparing the SAE 866A Method with the proposed SAE Method for the A-weighted and Perceived Noise metrics (ΔL_A and ΔL_{PN}).

The level-difference data (SAE 866A Method minus the proposed SAE Method) for the ΔL_A and ΔL_{PN} metrics obtained at the four temperature and humidity conditions were not included in Figures 6 and 7 because their spans are much larger than those of the first four sensitivity conditions. Over the four temperature and humidity conditions and the five altitudes up to 1200 m, the mean differences for ΔL_A and ΔL_{PN} were -0.08 dB and -0.07 dB, respectively, the maxima of the differences were 0.18 dB and 0.21 dB, the minima of the differences were -0.60 dB and -0.71 dB, and $\pm 2\sigma$ values (± 2 standard deviations) were 0.40 dB and 0.51 dB. For altitudes up to 7620 m, the mean and 2σ statistics of ΔL_A are -0.55 dB and ± 2.95 dB, respectively.

Presented in Appendix C, Figure C5 (a-d) are level-difference curves (SAE 866A Method minus the proposed SAE Method) for the calculated one-third octave-band source-to-receiver atmospheric

absorption adjustment levels (50 Hz to 10 kHz) for each of the four selected temperature and humidity points. The mid-band attenuation level at 7620 meters for the 10 kHz band (band 40) is included at the top of each figure as a point of reference.

The one-third octave-band level-difference data of Figure C5 (a-d), for the four temperature and humidity points, were combined and are presented in Figure 16. Shown in Figure 16 is the level-difference over the altitude range of 75 to 7620 meters. Mid-band level data is presented in Figure 17. Included are the mean, maximum, minimum, and $\pm 2\sigma$ values (± 2 standard deviations) calculated across the 24 one-third octave-bands.

RMSE statistics for the one-third octave-band level-difference data of Figures 16 and 17 are included in the fifth results column of Table 4. Although not shown herein, the differences comparing the SAE 866A Method directly to the Exact Method were found to be in the same range as the differences shown in Table 4 for the SAE 866A Method minus proposed SAE Method condition. Thus, it is concluded that the proposed SAE Method is in better agreement with the Exact Method than is the SAE 866A Method.

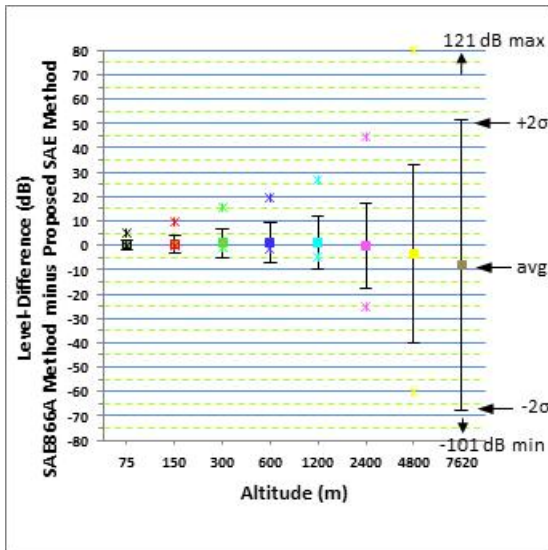


Figure 16: One-Third Octave-Band Level-Difference Data
 Combined Data – 4 T/H Points
 SAE 866A Method minus proposed SAE Method
 Static Pressure 101.325 kPa

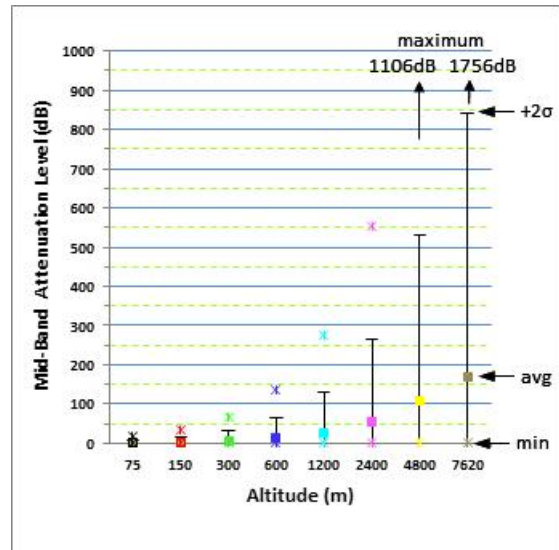


Figure 17: Mid-Band Level Data
 Combined Data – 4 T/H Points
 Static Pressure 101.325 kPa

4.6 Applicability to 20 kHz

In this section, the test spectrum was extended one octave to 20 kHz and the proposed SAE Method was again compared with the Exact Method. Extension of the test spectrum to the higher frequency bands was achieved by extrapolation. The slope of the original test spectrum remained a constant -1 dB per one-third octave-band over its last 16 bands, between 630 Hz and 10 kHz. Therefore, this slope was followed to extend the spectrum to 20 kHz.

For these comparisons, a homogenous atmosphere was assumed, i.e. temperature, humidity and pressure did not vary with altitude. Atmospheric pressure was set to a static, ISA sea level value of 101.325 kPa for the four temperature and humidity conditions.

Atmospheric absorption adjustments were made to the source one-third octave-band noise data at eight altitudes from 75 to 7620 meters at the four temperature and humidity conditions selected. Presented in Appendix C, Figure C6 (a-d) are level-difference curves (proposed SAE Method minus the Exact Method) for the calculated one-third octave-band source-to-receiver adjustment levels (for clarity, only curves for 100 Hz to 20 kHz are shown). The mid-band attenuation levels at 2400 meters, for the 10 kHz and 20 kHz bands (bands 40 and 43), are included at the top of each figure as a point of reference.

The one-third octave-band level-difference data of Figure C6 (a-d), for the four temperature and humidity points, were combined and are presented in Figure 18. The level-difference in Figure 18 is shown over the altitude range of 75 to 7620 meters. Though it produces the minimum level difference value, the attenuation calculated by the Exact Method at 7620 m altitude, 20 kHz frequency, 32 °C and 20%RH is so large that it exceeds EXCEL's limit of precision and, therefore, the minimum value of the level difference can be neither reported, nor factored into the standard deviation calculation. Mid-band level data are presented in Figure 19. Included are the mean, maximum, minimum, and $\pm 2\sigma$ values (± 2 standard deviations) calculated across the 24 one-third octave-bands (100 Hz to 20 kHz).

RMSE statistics for the one-third octave-band level-difference data of Figures 18 and 19 are included in the sixth results column of Table 4. Note that the level-differences for the extended frequency range compare favorably to the results in Section 4.1 (50 Hz-10 kHz).

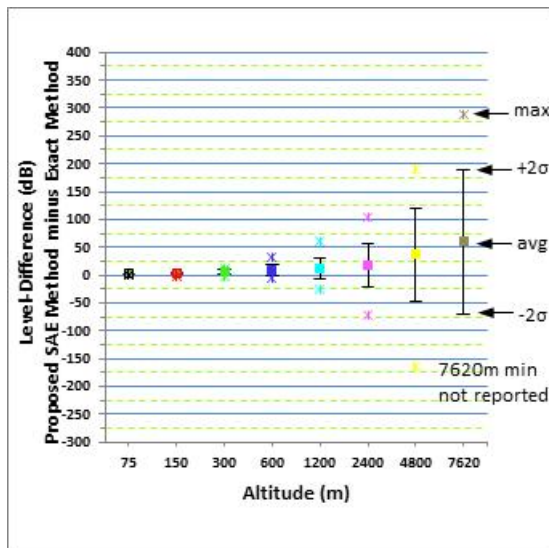


Figure 18: One-Third Octave-Band Level-Difference Data
Combined Data – 4 T/H Points
Proposed SAE Method minus Exact Method
100Hz to 20 kHz
Static Pressure 101.325 kPa

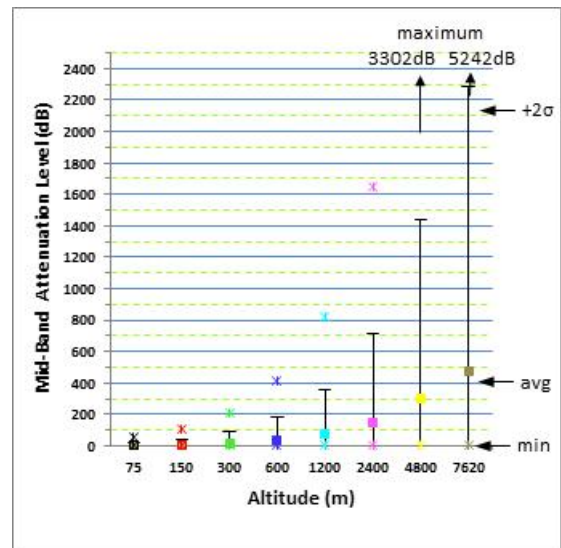


Figure 19: Mid-Band Level Data
Combined Data – 4 T/H Points
100 Hz to 20 kHz
Static Pressure 101.325 kPa

5. CONCLUSION

A new simplified procedure (the proposed SAE Method) is introduced for the calculation of atmospheric absorption for broadband sounds analyzed by one-third octave-band filters. The proposed method utilizes the accurate pure-tone sound absorption algorithms of the ISO/ANSI^{2,3} standards for predicting band-level attenuations and is recommended for mid-band attenuation up to 500 dB. The proposed SAE Method was evaluated under a variety of temperature and humidity conditions, atmospheric pressure conditions, filter shapes, and over the extended frequency range of 25 Hz to 20 kHz.

The proposed SAE Method was applied to representative commercial jet aircraft spectra over a range of altitudes from 75 to 7620 meters at the four selected temperature and humidity points. Two noise metrics, L_{PN} and L_A , were computed. These data were compared with similar data computed with the Exact Method. Level-difference results (Figures 6 and 7) over the altitude range of 75 to 1200 meters show the mean difference for both the ΔL_{PN} and ΔL_A descriptors is less than -0.07 dB with a 2σ range of less than ± 0.07 dB for the four conditions used in the evaluation (Section 4.1 to 4.4).

Reherman¹² et al. also performed a sensitivity analysis comparing the proposed SAE Method and SAE 866A procedure. Level differences (ΔL_A) for measured aircraft data processed for distances between 120 and 1200 meters showed a mean difference of less than 0.2 dB with a 1σ range of less than ± 0.5 dB.

Table 4 summarizes the one-third octave-band level-difference data observed in Sections 4.1 to 4.6. The proposed SAE Method is compared directly against the Exact Method under a variety of conditions using a representative aircraft noise spectrum over an altitude range of 75 to 7620 meters. Table 4 includes data for the frequency range of 50 Hz to 20 kHz and for the Approximate Method.

The data in Table 4 shows the proposed SAE Method to be more accurate than the SAE 866A Method, as well as the Approximate Method at the larger range of mid-band attenuation levels for which it is valid, as compared to the Exact Method. Unlike the SAE 866A Method, the proposed SAE Method can take into account the effects of changes in atmospheric pressure and unlike the Approximate Method it is useable to mid-band attenuation levels up to 500 dB. Lapsed pressure had little effect on the proposed SAE Method's performance as did the change in filter characteristics. Extending the frequency range to 20 kHz also had minimal effect.

The proposed SAE Method is seen to be useable to mid-band levels up to 500 dB with root mean square errors of less than 3% (of the mid-band attenuation level) to 100 dB and 4% to 500 dB. The proposed SAE Method is as easy to apply as the SAE 866A Method, can take into account the effects of changes in atmospheric pressure and is accurate from 25 Hz through 20 kHz.

It is recommended that the proposed SAE Method, presented herein, combined with the ISO/ANSI^{2,3} standards, replace the SAE 866A Method for the computation of attenuation by atmospheric absorption of broadband sounds when analyzed by one-third octave-band filters and that it be adopted as the method of choice in Part 36 and its international counterpart ICAO Annex 16⁵.

6. ACKNOWLEDGEMENTS

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¹⁰ Kneser, H.D., *Interpretation of the Anomalous Sound Absorption in Air and Oxygen in Terms of Molecular Collisions*, Acoust. Soc. Of Am., 5, 1933, pp. 122-126.

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APPENDIX A. Proposed SAE Method Performance vs. Mid-Band Attenuation Level: Broadband Source, Four Spectral Shapes

Level difference data (proposed SAE Method minus Exact Method) are shown in Figures A1 to A4 for each of the eleven temperature and humidity points plotted as a function of mid-band attenuation level. Figures A1a, A2a, A3a, and A4a show results for the 0, -2, +5, and -5 dB spectral data slopes, respectively. Figures A1b to A4b zoom in on the first 60 dB of mid-band attenuation data from Figures A1a to A4a.

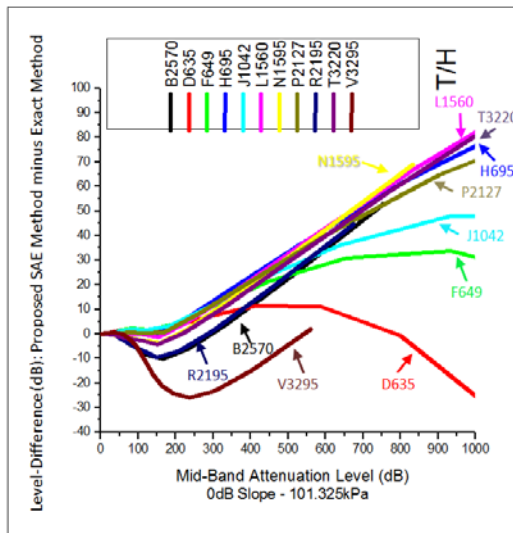


Figure A1a – Level Difference vs. Mid-Band Attenuation Level
Proposed SAE Method minus Exact Method
0 dB SLOPE – Static 101.325 kPa

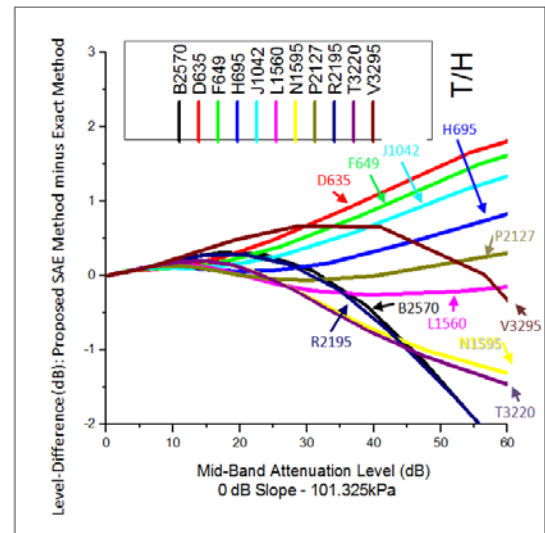


Figure A1b – Expanded Scale
Level Diff. vs. Mid-Band Attenuation Level
0 dB SLOPE – Static 101.325 kPa
(see Figure A1a)

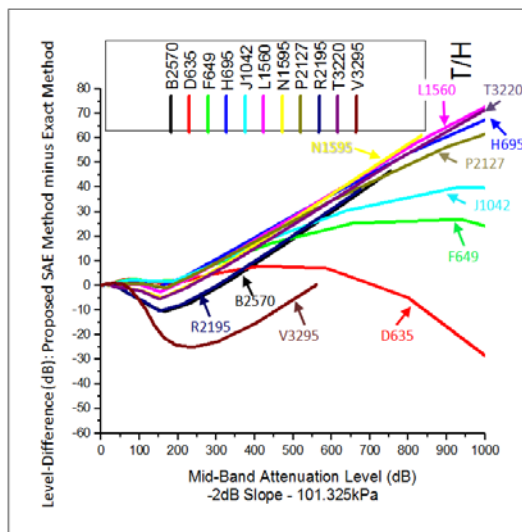


Figure A2a – Level Difference vs. Mid-Band Attenuation Level
Proposed SAE Method minus Exact Method
-2 dB SLOPE – Static 101.325 kPa

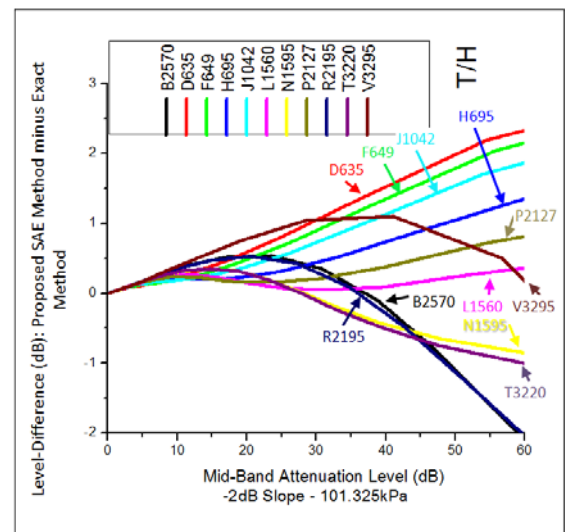
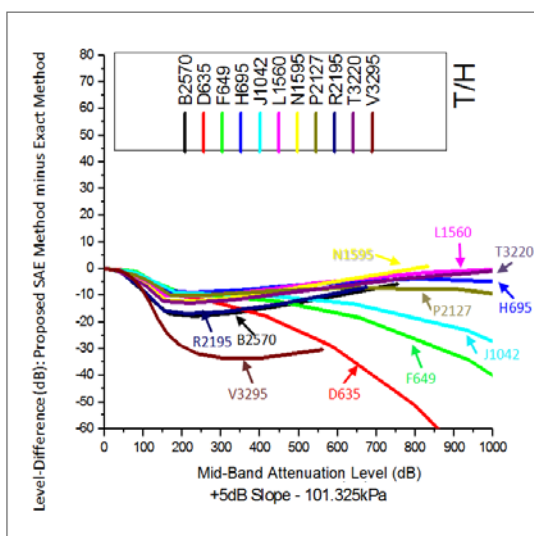


Figure A2b – Expanded Scale
Level Diff. vs. Mid-Band Attenuation Level
-2 dB SLOPE – Static 101.325 kPa
(see Figure A2a)



**Figure A3a – Level Difference vs.
Mid-Band Attenuation Level**
Proposed SAE Method minus Exact Method
+5 dB SLOPE – Static 101.325 kPa

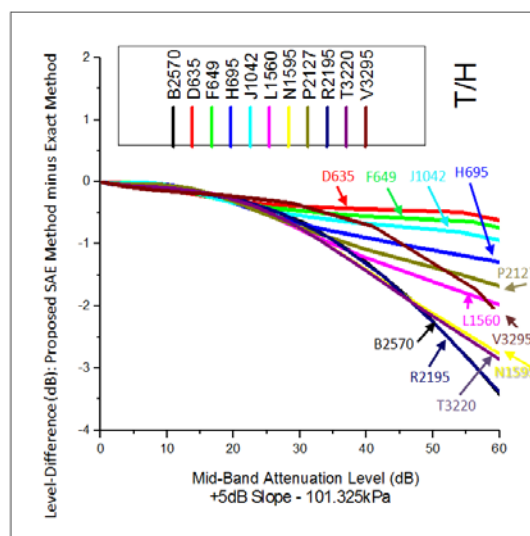
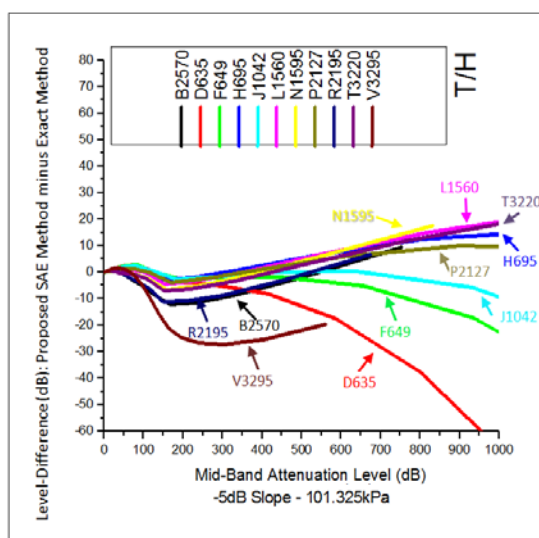


Figure A3b – Expanded Scale
Level Diff. vs. Mid-Band Attenuation Level
+5 dB SLOPE – Static 101.325 kPa
(see Figure A3a)



**Figure A4a – Level Difference vs.
Mid-Band Attenuation Level**
Proposed SAE Method minus Exact Method
-5 dB SLOPE – Static 101.325 kPa

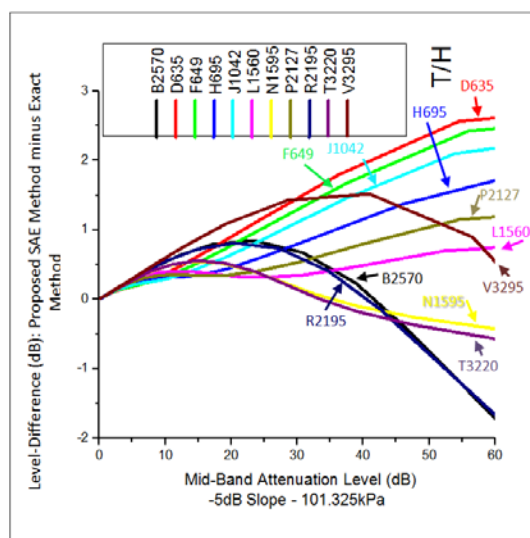


Figure A4b – Expanded Scale
Level Diff. vs. Mid-Band Attenuation Level
-5 dB SLOPE – Static 101.325 kPa
(see Figure A4a)

APPENDIX B. Proposed SAE Method Performance over Altitude: Broadband Source, Five Sensitivity Conditions

Presented in Tables B1 to B4 is the source-to-receiver level-difference data obtained comparing the proposed SAE Method with the Exact Method for the A-weighted and Perceived Noise metrics (ΔL_A and ΔL_{PN}). Table B5 shows the level-difference data obtained comparing the SAE 866A Method with the proposed SAE Method. Atmospheric absorption adjustments were made to the source one-third octave-band noise data at eight altitudes from 75 to 7620 meters at four selected temperature and humidity points. For entries marked by an asterisk, an insufficient number of one-third octave-bands fall within the acceptable level range due to the significant attenuation at large propagation distances. Therefore, the perceived level noise metric could not be calculated.

Table B1. Source-to-Receiver Level-Difference vs. Altitude
Proposed SAE Method minus Exact Method
Static 101.325 kPa Pressure - Homogeneous Temperature/Humidity

Altitude	Temp/Hum		Temp/Hum		Temp/Hum		Temp/Hum	
	<u>32°C,</u>	<u>20%RH</u>	<u>32°C,</u>	<u>95%RH</u>	<u>6°C,</u>	<u>49%RH</u>	<u>25°C,</u>	<u>70%RH</u>
<u>Altitude</u>	<u>ΔL_A</u>	<u>ΔL_{PN}</u>	<u>ΔL_A</u>	<u>ΔL_{PN}</u>	<u>ΔL_A</u>	<u>ΔL_{PN}</u>	<u>ΔL_A</u>	<u>ΔL_{PN}</u>
meters	dB	dB	dB	dB	dB	dB	dB	dB
75	-0.02	-0.04	-0.01	-0.02	-0.01	-0.04	-0.01	-0.02
150	-0.02	-0.07	-0.03	-0.04	-0.02	-0.04	-0.02	-0.04
300	-0.04	-0.11	-0.04	-0.05	-0.03	-0.06	-0.04	-0.05
600	-0.06	-0.09	-0.06	-0.06	-0.05	-0.08	-0.06	-0.06
1200	-0.09	-0.12	-0.07	-0.08	-0.07	-0.09	-0.08	-0.09
2400	-0.10	N/A*	-0.09	N/A*	-0.09	N/A*	-0.10	N/A*
4800	-0.10	N/A*	-0.11	N/A*	-0.11	N/A*	-0.12	N/A*
7620	-0.09	N/A*	-0.13	N/A*	-0.12	N/A*	-0.13	N/A*

Table B2. Source-to-Receiver Level-Difference vs. Altitude
Proposed SAE Method minus Exact Method
Lapsed Pressure @ Altitude - Homogeneous Temperature/Humidity

<u>Altitude</u>	<u>Temp/Hum</u> <u>32°C,</u> <u>20%RH</u>		<u>Temp/Hum</u> <u>32°C,</u> <u>95%RH</u>		<u>Temp/Hum</u> <u>6°C,</u> <u>49%RH</u>		<u>Temp/Hum</u> <u>25°C,</u> <u>70%RH</u>	
	<u>ΔL_A</u>	<u>ΔL_{PN}</u>	<u>ΔL_A</u>	<u>ΔL_{PN}</u>	<u>ΔL_A</u>	<u>ΔL_{PN}</u>	<u>ΔL_A</u>	<u>ΔL_{PN}</u>
meters	dB	dB	dB	dB	dB	dB	dB	dB
75	-0.02	-0.04	-0.01	-0.02	-0.01	-0.04	-0.01	-0.02
150	-0.02	-0.07	-0.03	-0.04	-0.02	-0.04	-0.02	-0.04
300	-0.04	-0.07	-0.04	-0.05	-0.03	-0.06	-0.04	-0.05
600	-0.06	-0.09	-0.06	-0.06	-0.05	-0.08	-0.06	-0.07
1200	-0.09	-0.12	-0.07	-0.08	-0.06	-0.09	-0.08	-0.09
2400	-0.10	N/A*	-0.09	N/A*	-0.09	N/A*	-0.10	N/A*
4800	-0.10	N/A*	-0.11	N/A*	-0.11	N/A*	-0.12	N/A*
7620	-0.09	N/A*	-0.13	N/A*	-0.12	N/A*	-0.13	N/A*

Table B3. Source-to-Receiver Level-Difference vs. Altitude
Proposed SAE Method minus Exact Method
30 m Layers w/ Lapsed Pressure - Homogeneous Temperature/Humidity

<u>Altitude</u>	<u>Temp/Hum</u> <u>32°C,</u> <u>20%RH</u>		<u>Temp/Hum</u> <u>32°C,</u> <u>95%RH</u>		<u>Temp/Hum</u> <u>6°C,</u> <u>49%RH</u>		<u>Temp/Hum</u> <u>25°C,</u> <u>70%RH</u>	
	<u>ΔL_A</u>	<u>ΔL_{PN}</u>	<u>ΔL_A</u>	<u>ΔL_{PN}</u>	<u>ΔL_A</u>	<u>ΔL_{PN}</u>	<u>ΔL_A</u>	<u>ΔL_{PN}</u>
meters	dB	dB	dB	dB	dB	dB	dB	dB
75	-0.01	-0.03	-0.02	-0.03	-0.01	-0.04	-0.01	-0.02
150	-0.02	-0.06	-0.03	-0.05	-0.02	-0.04	-0.02	-0.04
300	-0.04	-0.05	-0.04	-0.06	-0.03	-0.05	-0.04	-0.05
600	-0.07	-0.09	-0.06	-0.08	-0.04	-0.06	-0.06	-0.07
1200	-0.11	-0.13	-0.08	-0.12	-0.06	-0.07	-0.10	-0.12
2400	-0.17	N/A*	-0.08	N/A*	-0.09	N/A*	-0.13	N/A*
4800	-0.21	N/A*	-0.08	N/A*	-0.13	N/A*	-0.13	N/A*
7620	-0.21	N/A*	-0.08	N/A*	-0.17	N/A*	-0.11	N/A*

Table B4. Source-to-Receiver Level-Difference vs. Altitude

Proposed SAE Method minus Exact Method

Larson Davis Long Filter (LDLong)

Static 101.325 kPa Pressure - Homogeneous Temperature/Humidity

	<u>Temp/Hum</u> <u>32^oC,</u> <u>20%RH</u>		<u>Temp/Hum</u> <u>32^oC,</u> <u>95%RH</u>		<u>Temp/Hum</u> <u>6^oC,</u> <u>49%RH</u>		<u>Temp/Hum</u> <u>25^oC,</u> <u>70%RH</u>	
	<u>ΔL_A</u>	<u>ΔL_{PN}</u>	<u>ΔL_A</u>	<u>ΔL_{PN}</u>	<u>ΔL_A</u>	<u>ΔL_{PN}</u>	<u>ΔL_A</u>	<u>ΔL_{PN}</u>
Altitude								
meters	dB	dB	dB	dB	dB	dB	dB	dB
75	-0.01	-0.04	-0.01	-0.02	-0.01	-0.04	-0.01	-0.02
150	-0.02	-0.06	-0.03	-0.04	-0.02	-0.04	-0.02	-0.04
300	-0.04	-0.10	-0.04	-0.04	-0.03	-0.05	-0.04	-0.05
600	-0.06	-0.09	-0.05	-0.06	-0.05	-0.07	-0.06	-0.06
1200	-0.08	-0.11	-0.07	-0.07	-0.06	-0.08	-0.08	-0.08
2400	-0.10	N/A*	-0.09	N/A*	-0.09	N/A*	-0.09	N/A*
4800	-0.09	N/A*	-0.11	N/A*	-0.11	N/A*	-0.11	N/A*
7620	-0.09	N/A*	-0.12	N/A*	-0.11	N/A*	-0.12	N/A*

Table B5. Source-to-Receiver Level-Difference vs. Altitude

SAE ARP 866A-1975 Method minus Proposed SAE Method

Static 101.325 kPa Pressure - Homogeneous Temperature/Humidity

	<u>Temp/Hum</u> <u>32^oC,</u> <u>20%RH</u>		<u>Temp/Hum</u> <u>32^oC,</u> <u>95%RH</u>		<u>Temp/Hum</u> <u>6^oC,</u> <u>49%RH</u>		<u>Temp/Hum</u> <u>25^oC,</u> <u>70%RH</u>	
	<u>ΔL_A</u>	<u>ΔL_{PN}</u>	<u>ΔL_A</u>	<u>ΔL_{PN}</u>	<u>ΔL_A</u>	<u>ΔL_{PN}</u>	<u>ΔL_A</u>	<u>ΔL_{PN}</u>
Altitude								
meters	dB	dB	dB	dB	dB	dB	dB	dB
75	0.00	0.13	0.02	0.03	-0.01	0.21	0.00	0.02
150	-0.02	0.07	0.03	0.04	-0.08	0.21	-0.01	-0.02
300	-0.03	0.15	0.03	-0.11	-0.20	0.07	-0.02	0.01
600	0.01	-0.04	-0.08	-0.23	-0.38	-0.29	0.01	-0.15
1200	0.18	0.18	-0.53	-0.71	-0.60	-0.62	0.07	-0.33
2400	0.60	N/A*	-1.72	N/A*	-0.87	N/A*	-0.06	N/A*
4800	1.18	N/A*	-4.09	N/A*	-1.12	N/A*	-0.98	N/A*
7620	1.24	N/A*	-6.55	N/A*	-1.23	N/A*	-2.39	N/A*

APPENDIX C. Proposed SAE Method Performance over Altitude: One-Third Octave-Bands, Six Sensitivity Conditions

Figures C1 (a-d) through C6 (a-d) in this Appendix contain level-difference curves for source-to-receiver adjustment levels, for each of the four selected temperature and humidity points, under each of the six sensitivity conditions. Presented in Figures C1 (a-d) through C4 (a-d) are level-difference curves for one-third octave-bands 50 Hz to 10 kHz, calculated as proposed SAE Method minus Exact Method. Figures C5 (a-d) present level-difference curves for one-third octave-bands 50 Hz to 10 kHz calculated as SAE 866A Method minus proposed SAE Method. Figure C6 (a-d) show level-difference curves for one-third octave-bands 100Hz to 20 kHz, calculated as proposed SAE Method minus Exact Method.

Band 24 – Mid-Band Attenuation @ 7620 m =1755.9 dB

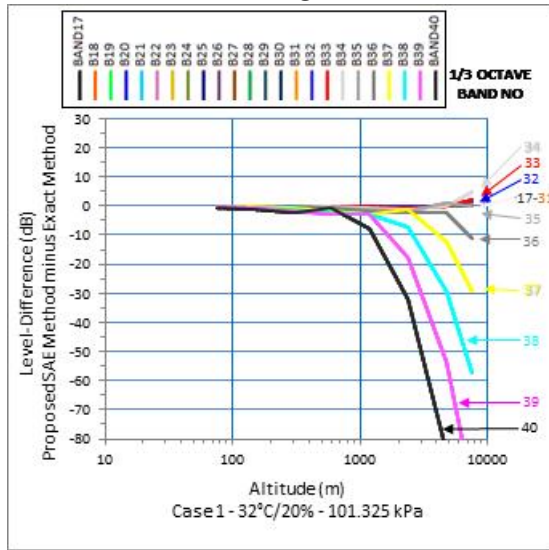


Figure C1a – Level-Difference vs. Altitude
Proposed SAE Method minus Exact Method
32°C, 20%RH, Static 101.325 kPa

Band 24 – Mid-Band Attenuation @ 7620 m =560.5

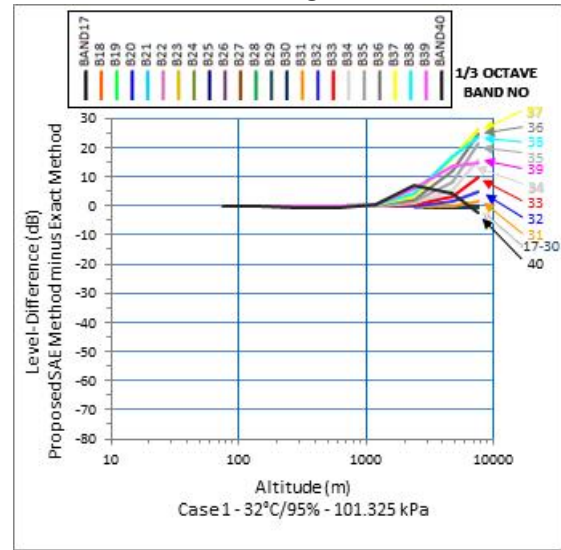


Figure C1b – Level-Difference vs. Altitude
Proposed SAE Method minus Exact Method
32°C, 95%RH, Static 101.325 kPa

Band 24 – Mid-Band Attenuation @ 7620 m =1690.9 dB

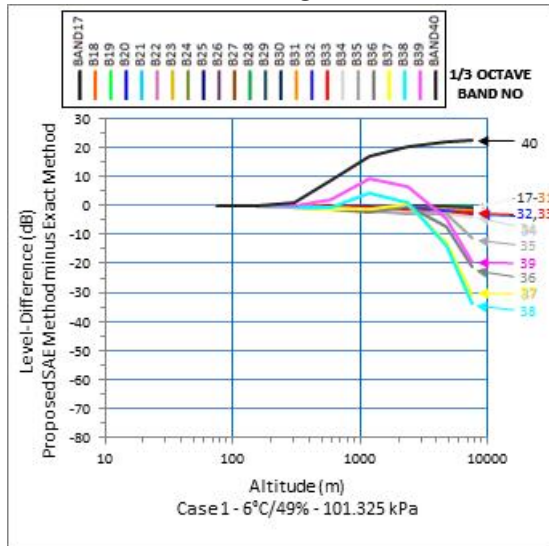


Figure C1c – Level-Difference vs. Altitude
Proposed SAE Method minus Exact Method
6°C, 49%RH, Static 101.325 kPa

Band 24 – Mid-Band Attenuation @ 7620 m =753.8 dB

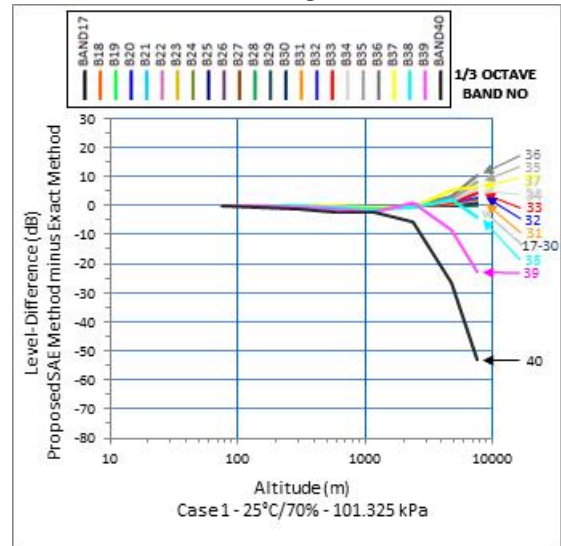


Figure C1d – Level-Difference vs. Altitude
Proposed SAE Method minus Exact Method
25°C, 70%RH, Static 101.325 kPa

Band 24 – Mid-Band Attenuation @ 7620 m =1727.2 dB

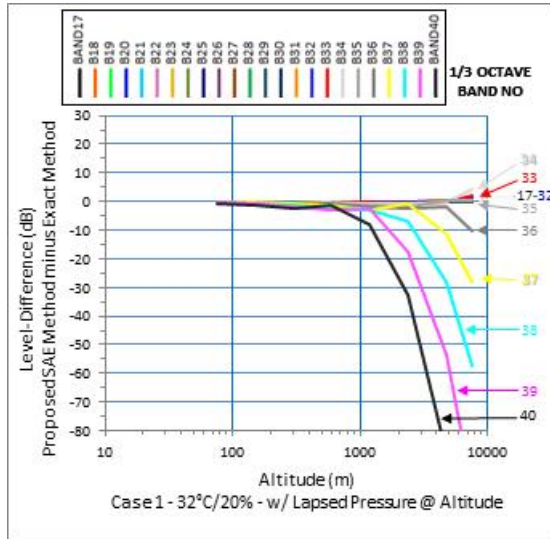


Figure C2a – Level-Difference vs. Altitude
Proposed SAE Method minus Exact Method
32°C, 20%RH, w/Lapsed Pressure

Band 24 – Mid-Band Attenuation @ 7620 m =734.6 dB

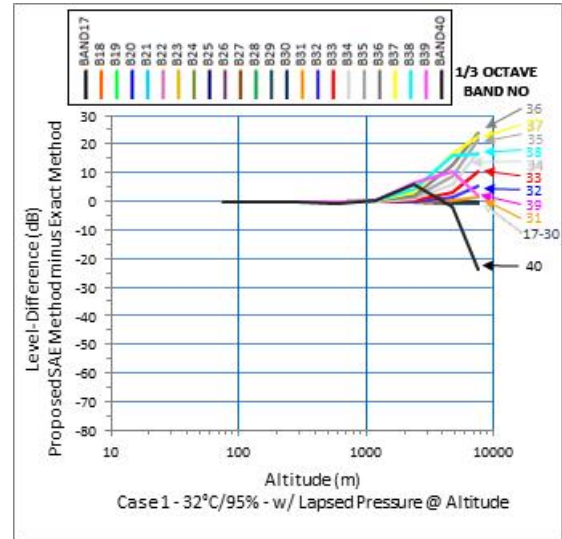


Figure C2b – Level-Difference vs. Altitude
Proposed SAE Method minus Exact Method
32°C, 95%RH, w/Lapsed Pressure

Band 24 – Mid-Band Attenuation @ 7620 m =1792.2 dB

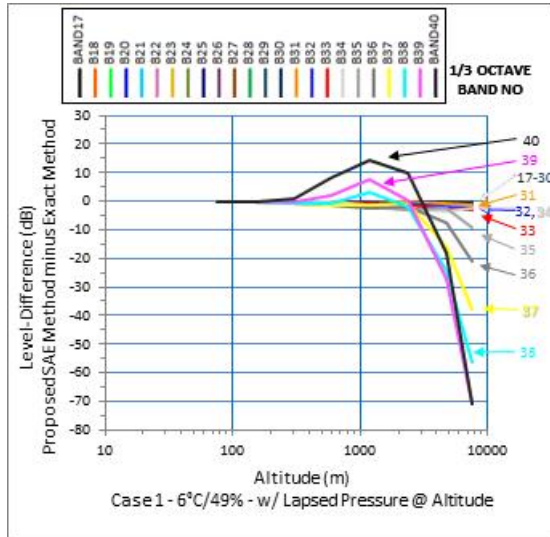


Figure C2c – Level-Difference vs. Altitude
Proposed SAE Method minus Exact Method
6°C, 49%RH, w/Lapsed Pressure

Band 24 – Mid-Band Attenuation @ 7620 m =891.3 dB

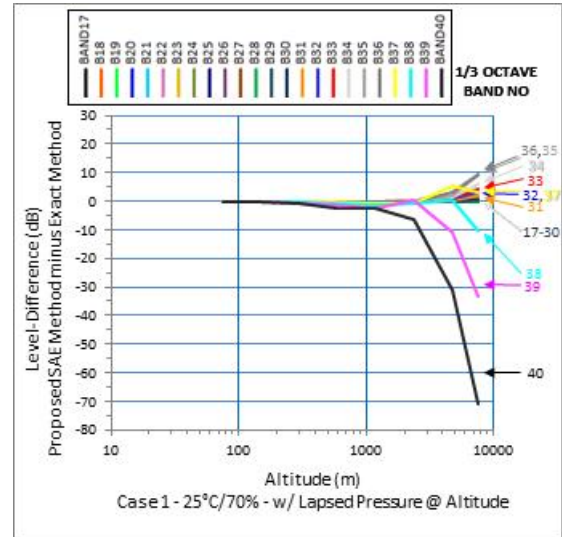


Figure C2d – Level-Difference vs. Altitude
Proposed SAE Method minus Exact Method
25°C, 70%RH, w/Lapsed Pressure

Band 24 – Mid-Band Attenuation @ 7620 m =1713.4 dB

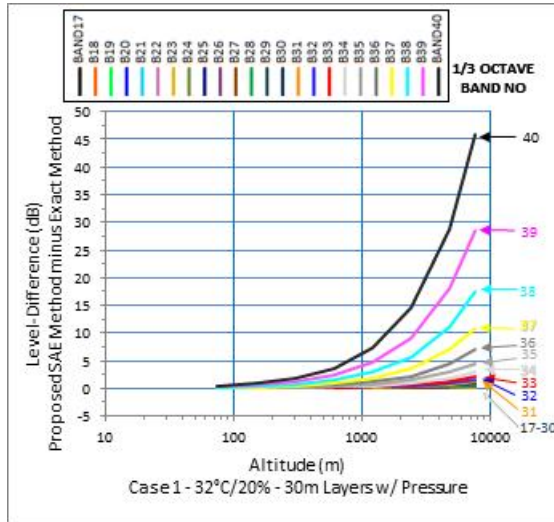


Figure C3a – Level-Difference vs. Altitude
Proposed SAE Method minus Exact Method
32°C, 20%RH, 30-m Layers w/Pressure

Band 24 – Mid-Band Attenuation @ 7620 m =632.3 dB

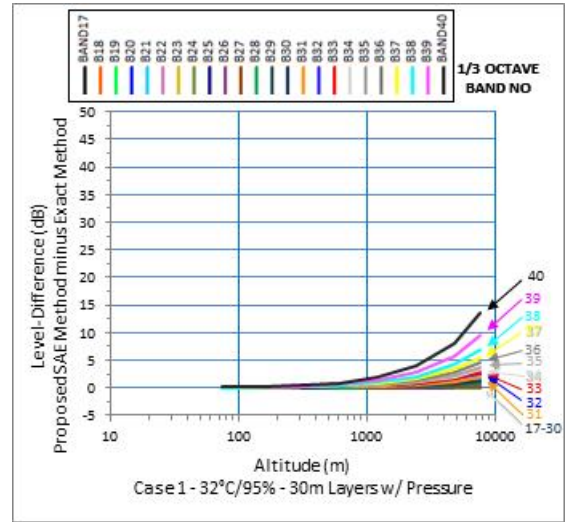


Figure C3b – Level-Difference vs. Altitude
Proposed SAE Method minus Exact Method
32°C, 95%RH, 30-m Layers w/Pressure

Band 24 – Mid-Band Attenuation @ 7620 m =1734.9 dB

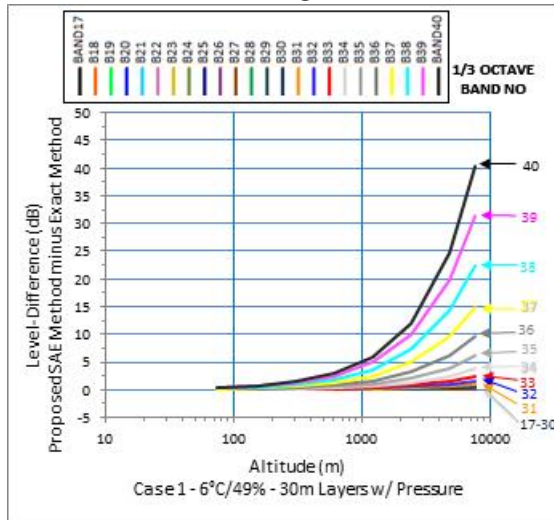


Figure C3c – Level-Difference vs. Altitude
Proposed SAE Method minus Exact Method
6°C, 49%RH, 30-m Layers w/Pressure

Band 24 – Mid-Band Attenuation @ 7620 m =804.9 dB

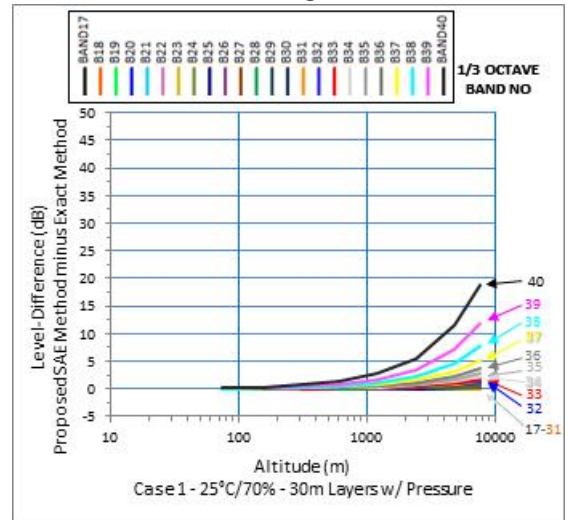


Figure C3d – Level-Difference vs. Altitude
Proposed SAE Method minus Exact Method
25°C, 70%RH, 30-m Layers w/Pressure

Band 24 – Mid-Band Attenuation @ 7620 m =1755.9 dB

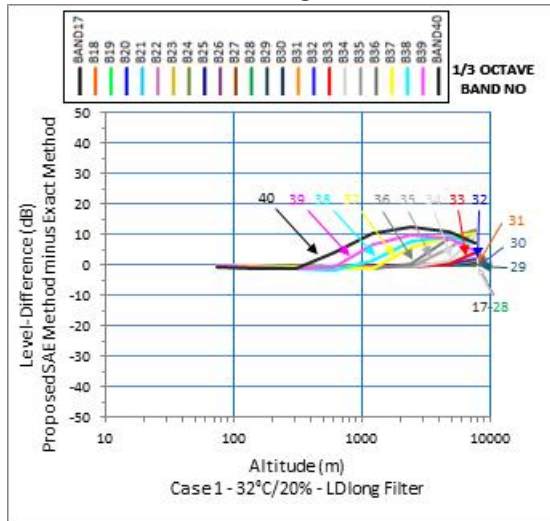


Figure C4a – Level-Difference vs. Altitude
Proposed SAE Method minus Exact Method
LD_{Long} Filter
32°C, 20%RH, Static 101.325 kPa

Band 24 – Mid-Band Attenuation @ 7620 m =560.5 dB

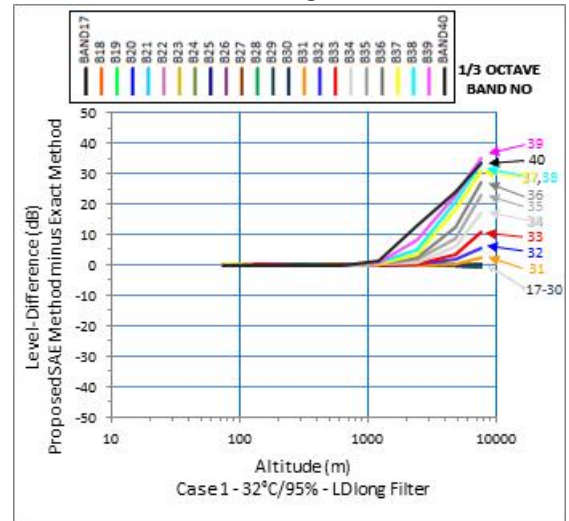


Figure C4b – Level-Difference vs. Altitude
Proposed SAE Method minus Exact Method
LD_{Long} Filter
32°C, 95%RH, Static 101.325 kPa

Band 24 – Mid-Band Attenuation @ 7620 m =1690.9 dB

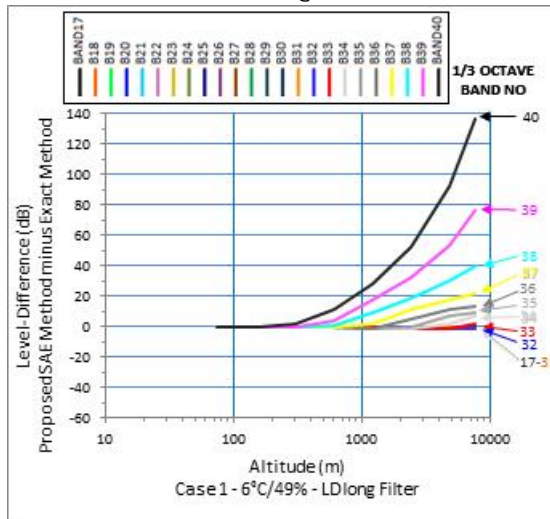


Figure C4c – Level-Difference vs. Altitude
Proposed SAE Method minus Exact Method
LD_{Long} Filter
6°C, 49%RH, Static 101.325 kPa

Band 24 – Mid-Band Attenuation @ 7620 m =753.8 dB

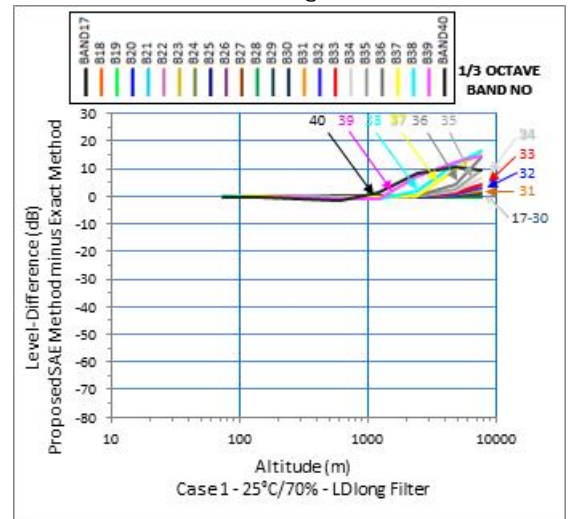


Figure C4d – Level-Difference vs. Altitude
Proposed SAE Method minus Exact Method
LD_{Long} Filter
25°C, 70%RH, Static 101.325 kPa

Band 24 – Mid-Band Attenuation @ 7620 m =1755.9 dB

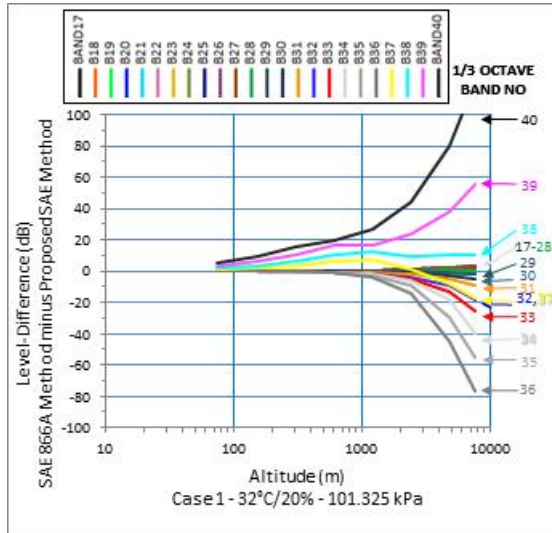


Figure C5a – Level-Difference vs. Altitude
SAE 866A Method minus Proposed SAE Method
32°C, 20%RH, Static 101.325 kPa

Band 24 – Mid-Band Attenuation @ 7620 m =560.5 dB

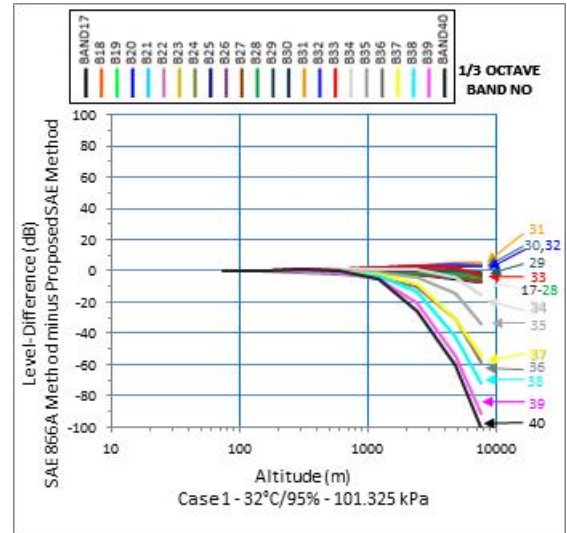


Figure C5b – Level-Difference vs. Altitude
SAE 866A Method minus Proposed SAE Method
32°C, 95%RH, Static 101.325 kPa

Band 24 – Mid-Band Attenuation @ 7620 m =1690.9 dB

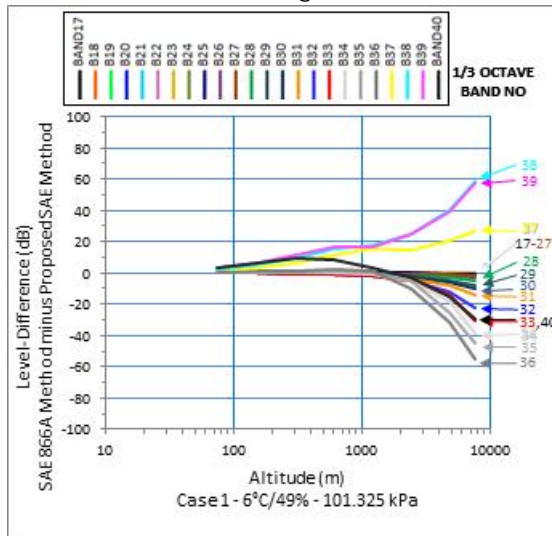


Figure C5c – Level-Difference vs. Altitude
SAE 866A Method minus Proposed SAE Method
6°C, 49%RH, Static 101.325 kPa

Band 24 – Mid-Band Attenuation @ 7620 m =753.8 dB

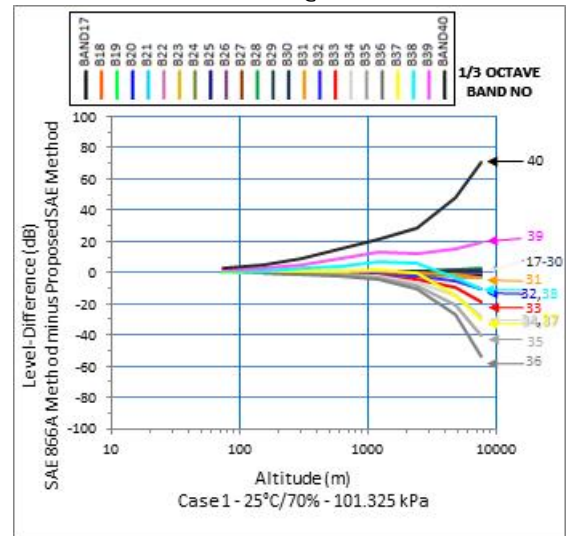


Figure C5d – Level-Difference vs. Altitude
SAE 866A Method minus Proposed SAE Method
25°C, 70%RH, Static 101.325 kPa

Band 24 – Mid-Band Attenuation @ 2400 m = 553.0 dB
 Band 27 – Mid-Band Attenuation @ 2400 m = 1651.1 dB

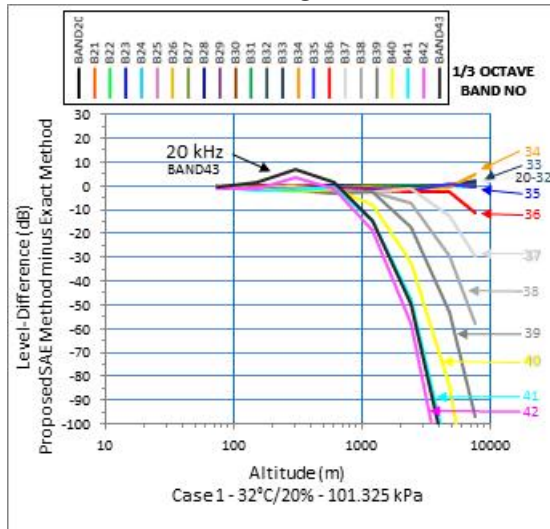


Figure C6a – Level-Difference vs. Altitude
 Proposed SAE Method minus Exact Method
 100 Hz to 20 kHz
 32°C, 20%RH, Static 101.325 kPa

Band 24 – Mid-Band Attenuation @ 2400 m = 176.5 dB
 Band 27 – Mid-Band Attenuation @ 2400 m = 569.1 dB

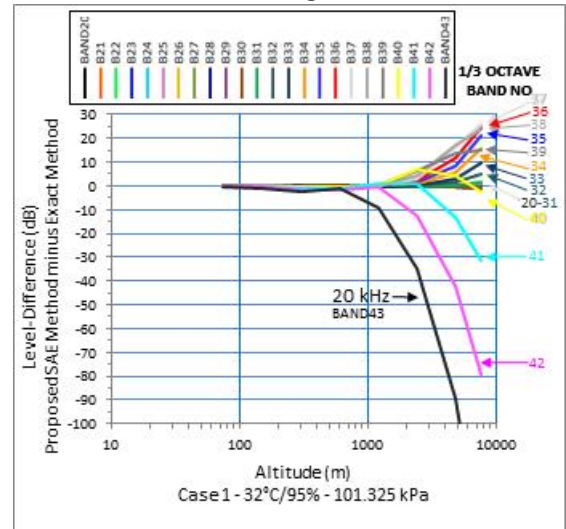


Figure C6b – Level-Difference vs. Altitude
 Proposed SAE Method minus Exact Method
 100 Hz to 20 kHz
 32°C, 95%RH, Static 101.325 kPa

Band 24 – Mid-Band Attenuation @ 2400 m = 532.6 dB
 Band 27 – Mid-Band Attenuation @ 2400 m = 950.9 dB

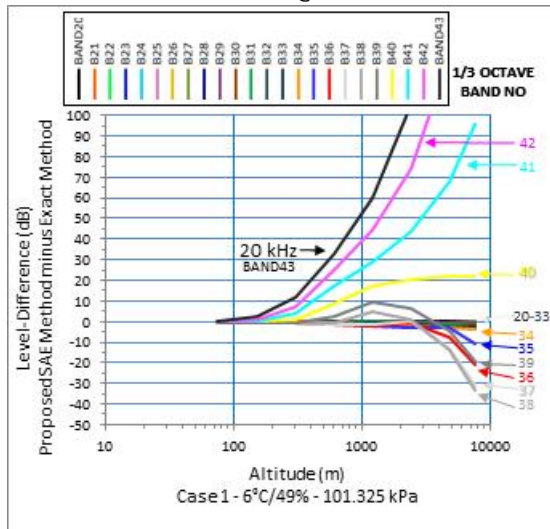


Figure C6c – Level-Difference vs. Altitude
 Proposed SAE Method minus Exact Method
 100 Hz to 20 kHz
 6°C, 49%RH, Static 101.325 kPa

Band 24 – Mid-Band Attenuation @ 2400 m = 237.4 dB
 Band 27 – Mid-Band Attenuation @ 2400 m = 857.8 dB

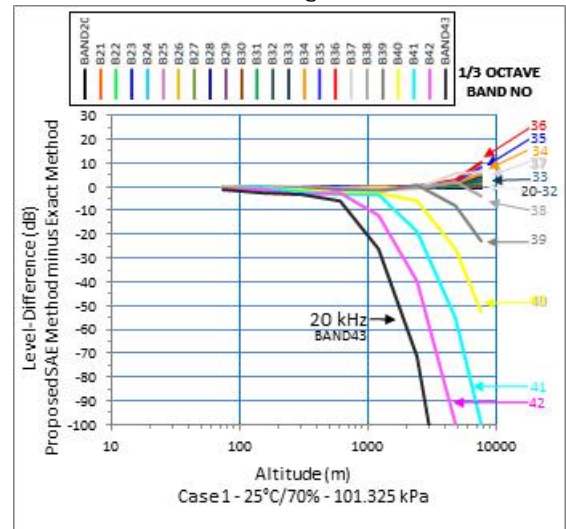


Figure C6d – Level-Difference vs. Altitude
 Proposed SAE Method minus Exact Method
 100 Hz to 20 kHz
 25°C, 70%RH, Static 101.325 kPa

APPENDIX D. Sensitivity Analysis Comparing an Earlier Form of the Proposed SAE Method SAE ARP 866A-1975 Procedure

Reherman, Roof, and Fleming performed a sensitivity analysis comparing the SAE ARP 866A-1975 procedures with an earlier form of the proposed SAE Method¹². Because the proposed SAE Method evolved over time, subtle differences exist between the current proposed SAE Method and the form used by Reherman et al. However, sensitivities were checked (but not fully redone) to ensure differences between the results of the current and earlier forms were negligible..

Thirteen takeoff and two approach spectra for fixed-wing aircraft, as well as one takeoff, two approaches, and three flyover spectra for helicopters, were utilized in the Reherman et al. analysis. A summary of sound level difference data processed at five distances between 120 and 1200 meters is presented in Figure D1. As-measured spectra were corrected to the distances for five different temperature and humidity combinations within the Part 36 allowable test window, to generate values of adjusted spectra and associated A-weighted maximum sound levels (L_{ASmx}). Atmospheric pressure was set to a fixed, ISA sea level value of 101.325 kPa. Additional distances (not shown), covering the ten standard distances between 60 and 7620 meters, were also run.

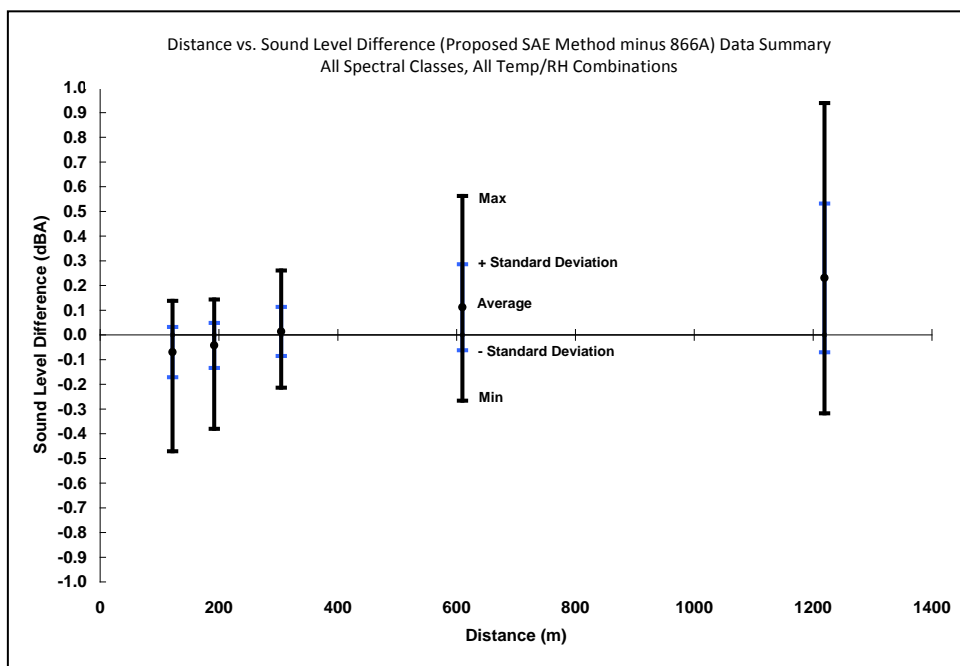


Figure D1 – Difference Data Summary
Proposed SAE Method minus SAE ARP 866A-1975 Procedure